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## Topic 1: Introduction to Fluid Mechanics

### **Introduction**

Fluid Mechanics is that discipline within the broad field of applied mechanics concerned of the behavior of liquids and gases at rest or in motion. This field is vast that may vary from the study of blow flow in the capillaries to the flow of crude oil across Alaska. Is almost certain that every system you will work in the future required a good understanding of fluid mechanics.

### **Section 1.1 - Some Characteristics of Fluids**

- One of the major differences between fluids and solids is how they deform under the action of an external load. Solids has densely spaced molecules with large intermolecular cohesive forces that allow it to maintain its shape, and to not be easily deformed. Otherwise fluids have smaller intermolecular forces which give more flexibility and the molecules have more freedom of movement. Thus they can be easily deformed.
- A **fluid** can be defined as a substance that could be deformed continuously when acted on by a shearing stress of any magnitude.
- A shearing stress, which has units of force per unit area, is created whenever a tangential force acts on a surface. The dimension of this kind of pressure is force per unit area.

## Section 1.2 - Dimensions, Dimensional Homogeneity, and Units

- Fluids characteristics can be described qualitatively in terms of certain basic quantities such as:

Mass	Length	Time	Temperature	Force
M	L	T	$\theta$	F

- Basic quantities can be used to provide a qualitative description of any other secondary quantities such as area =  $(L^2)$ , velocity =  $(L/T)$ , density =  $(M/L^3)$  and others.
- All theoretical derived equations are dimensionally homogeneous, which means that the dimensions of the left side of the equation must be the same dimensions as those on the right side.
  - a) Restricted homogeneous equations: equations that are restricted to a particular system of units.
  - b) general homogeneous equations: equations valid in any system of units
- There exist some **System of Units**:
  - ✓ **British Gravitational (BG) System**, where the unit of length is the foot (ft), the time is in second (s), the force unit is the pound (lb), and the temperature is the degree Fahrenheit ( $^{\circ}F$ ) or the absolute one is the degree Rankine ( $^{\circ}R$ ), where  $^{\circ}R = ^{\circ}F + 459.67$ , and the mass unit is called the slug.  
 $1 \text{ slug} = 32.174 \text{ lbm}$
  - ✓ **International System (SI)**, where the unit of length is the meter (m), time unit is the second (s), mass unit is the kilogram (kg) and the temperature unit is the Kelvin (K).  $K = ^{\circ}C + 273.15$ , where  $^{\circ}C$  is the temperature in degree Celsius. Force unit:  $1N = (1 \text{ kg})(1 \text{ m/s}^2)$ ; Work unit:  $1 \text{ Joule (J)} = 1 \text{ N}\cdot\text{m}$ ; Power Unit:  $1 \text{ Watt (W)} = 1 \text{ J/s}$ .

**TABLE 1.2**  
**Prefixes for SI Units**

Factor by Which Unit Is Multiplied	Prefix	Symbol
$10^{12}$	tera	T
$10^9$	giga	G
$10^6$	mega	M
$10^3$	kilo	k
$10^2$	hecto	h
10	deka	da
$10^{-1}$	deci	d
$10^{-2}$	centi	c
$10^{-3}$	milli	m
$10^{-6}$	micro	$\mu$
$10^{-9}$	nano	n
$10^{-12}$	pico	p
$10^{-15}$	femto	f
$10^{-18}$	atto	a

- ✓ **English Engineering (EE) System**, where the unit of mass is the pound mass (lbm), and the unit of force is the pound (lb or lbf). The unit of length is the foot, time is seconds and the absolute temperature is the °R.
  - ✓  $g_c$  is the constant of proportionality which allows us to define units for both force and mass.

$$g_c = \frac{(1 \text{ lbm}) \left( 32.174 \frac{\text{ft}}{\text{s}^2} \right)}{1 \text{ lb}}$$

### **Section 1.3 - Analysis of fluid Behavior**

The study of fluid mechanics involves the same fundamental law you have encountered in physics and other mechanic courses. Fluid mechanics is subdivided into fluid statics, in which the fluid is at rest, and fluid dynamics, in which the fluid is moving.

### **Section 1.4 - Measures of fluid mass and weight**

**a) Density ( $\rho$ ):**

The density of a fluid, designates by the Greek letter  $\rho$ , is defined as mass per unit of volume. Density is typically used to characterize the mass of a fluid system. The value of density can vary between fluids, but for liquids, variations in pressure and

temperature generally have only small effects on the value of  $\rho$ . Unlike liquids, the density of a gas is strongly influenced by both pressure and temperature.

$$\rho = \frac{m}{V}$$

**b) Specific volume ( $v$ ):**

It's the reciprocal of the density. It's not commonly used in fluid mechanics but used in thermodynamics.

$$v = \frac{1}{\rho}$$

**c) Specific Weight ( $\gamma$ ):**

It's designated by the Greek letter  $\gamma$  and is defined as its weight per unit of volume. The specific weight is related to the density by the following equation  $\gamma = \rho g$  where  $g$  is the local acceleration of gravity.

**d) Specific Gravity (SG):**

Designated as SG, is defined as the ratio of the density of the fluid to the density of the water at some specific temperature. Usually the temperature is 4<sup>0</sup>C.

$$SG = \frac{\rho}{\rho_{H2O @ 4^{\circ}C}}$$

Note: The properties of density and specific gravity are measures of the “heaviness” of a fluid, so they don't tell us anything about how fluids deform or flow.

## **Section 1.5 - Ideal Gas Law**

Gases are highly compressible compared to liquids, with changes in gases density directly related to changes in pressure and temperature, through the equation  $\mathbf{P = \rho RT}$ , where P is the absolute pressure,  $\rho$  the density, T the absolute temperature and R is a gas constant. It is commonly termed the ideal or perfect gas law, or the equation of state for an ideal gas. It

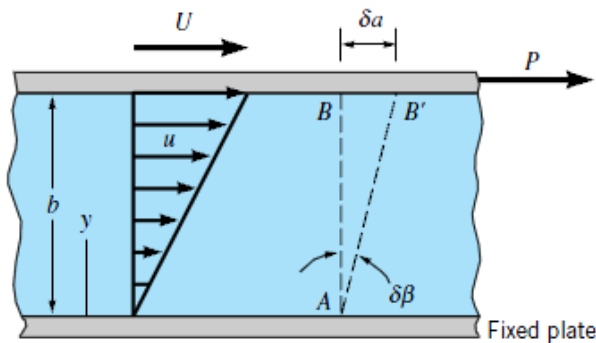
approximates the behavior of real gases under normal conditions when the gases are not approaching liquefaction.

**Pressure-** In fluid at rest pressure is defined as the normal force per unit of area exerted on a plane surface. Pressure has the dimension  $F/L^2$ . The pressure in the ideal gas law must be expressed as an absolute pressure denoted (abs), which means that is measured relative to the absolute zero pressure. Measure the pressure relative to the atmospheric pressure is called the gage pressure. Thus the absolute pressure can be obtained from the gage adding the atmospheric local pressure.

### Section 1.6 - Viscosity

The properties of density and specific weight are measures of the “heaviness” of a fluid. It is clear, however, that these properties are not sufficient to uniquely characterize how fluids behave since two fluids (such as water and oil) can have approximately the same value of density but behave quite differently when flowing. There is apparently some additional property that is needed to describe the “fluidity” of the fluid.

To determine this additional property, consider a hypothetical experiment in which a material is placed between two very wide parallel plates. The bottom plate is rigidly fixed, but the upper plate is free to move. If a solid, such as steel, were placed between the two plates and loaded with the force  $P$  as shown, the top plate would be displaced through some small distance,  $\delta a$  (assuming the solid was mechanically attached to the plates). The vertical line  $AB$  would be rotated through the small angle,  $\delta\beta$ , to the new position  $AB'$ ".



■ **FIGURE 1.3** Behavior of a fluid placed between two parallel plates.

To resist the force P a shearing stress,  $\tau$ , would be developed at the plate, material interface. To develop equilibrium  $P = \tau A$  where A is the effective upper plate area. When a force P is applied to the upper plate, it will move with a velocity, U. If we inspect the fluid motion between the two plates we see that the fluid in contact with the upper plate moves with the plate velocity, U, while the fluid in contact with the bottom plate has a zero velocity. So the fluid between the plates moves with a velocity  $u = u(y)$ , which varies linearly in this case. A velocity gradient  $du/dy$  is developed in the fluids between the plates. In this particular case the velocity gradient is constant since  $du/dy = U/b$ , but in more complex flow situations this would not be true.

- The no-slip condition is the experimental observation that the fluid sticks to the solid boundaries. All fluids satisfy this condition.
- In a small time increment,  $\delta t$ , an imaginary vertical line AB would rotate through the angle,  $\delta\beta$ , so  $\tan\delta\beta \approx \delta\beta = \delta a/b$  where  $da = U dt$  and  $db = U dt/b$
- We note that in this case,  $db$  is a function not only of the force P (which governs U) but also of time. So we consider the rate at which  $db$  is changing and define the **rate of**

**shearing strain** as

$$\dot{\gamma} = \lim_{\delta t \rightarrow 0} \frac{\delta\beta}{\delta t} \quad \text{which is equal to} \quad \dot{\gamma} = \frac{U}{b} = \frac{du}{dy}$$

- When the shearing stress,  $\tau$ , is increased by increasing P ( $\tau = P/A$ ), the rate of shearing strain is increased in direct proportion, that is

$$\tau \propto \dot{\gamma} \quad \text{or} \quad \tau \propto du/dy$$

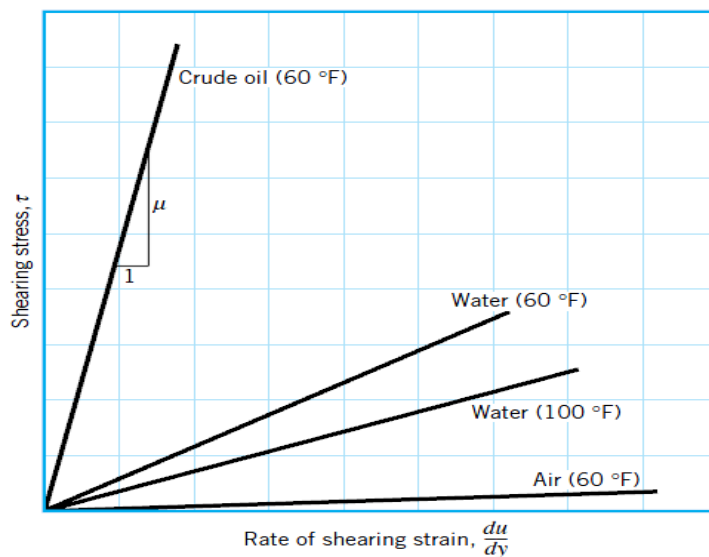
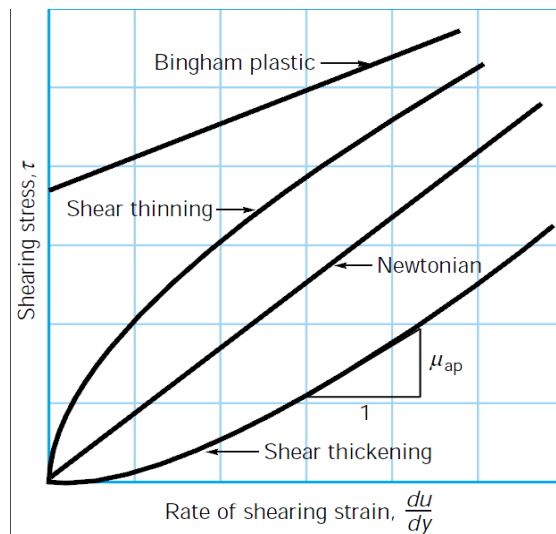
Therefore, for common fluids the shearing stress and rate of shearing strain (velocity gradient) can be related with the following equation:

$$\tau = -\mu \frac{du}{dy}$$

(Newton's Law of Viscosity)

where the constant of proportionality is called the **viscosity** of the fluid, which has a particular value for specific fluids.

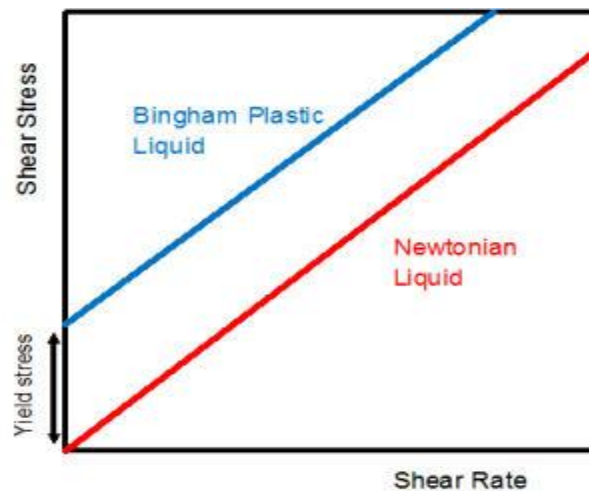
- Fluids for which the shearing stress is linearly related to the rate of shearing strain (angular deformation) are designated as Newtonian Fluid.
- Fluids with a molecular weight of less than 5000 are known as Newtonian Fluids. Polymeric liquids, suspensions, paste, slurries which are not described by Newton's Law of Viscosity are known as Non-Newtonian Fluids.
- Fluids for which the shearing stress is not linearly related to the rate of shearing strain are designated as Non-Newtonian Fluid.



■ **FIGURE 1.4** Linear variation of shearing stress with rate of shearing strain for common fluids.

- For shear thinning fluids the apparent viscosity decrease with increasing shear rate. The harder the fluid is shear the less viscous is become. Example: Latex, Paints.
- For shear thickening fluids the apparent viscosity increases with increasing the shear rate. The harder the fluid is shear the more viscous is become. Example: Water-Cornstarch mixture.
- The Bingham Plastic which is neither a fluid nor a solid, it is a viscoplastic material that behaves as a rigid body at low stresses but flows as a viscous fluid at high stress. It was named after Eugene C. Bingham proposes its mathematical form. Such materials can withstand a finite shear stress without motion, but once the yield stress is exceeded it flows like a fluid. Some examples are: toothpaste, mayonnaise, chocolate, drilling mud and mustard.

In a diagram of shear stress vs. Rate of shear stress is compared with a Newtonian fluid:



- The Dimensions of Viscosity are  $FT/L^2$ .
- Viscosity is very sensitive to temperature, but its dependence of pressure can be neglected.
- The effects of temperature on viscosity can be closely approximate using two empirical formulas. For gases the Sutherland equation can be express as:

$$\mu = \frac{CT^{3/2}}{T + S} \quad \text{where C and S are empirical constants}$$

- For liquids can be used the Andrade empirical equation:

$$\mu = De^{B/T} \quad \text{where D and B are constant and T is the absolute temperature}$$

- The ratio of viscosity with density is called the kinematic viscosity denominated with the Greek letter  $\nu$ .

$$\nu = \mu/\rho$$

- If a fluid is isotropic which means that it has no preferred direction we have the following equation derived from Newton's Law of Viscosity. (This reduces the number of "viscosity coefficients" from 81 to 2).

$$\tau_{ij} = A \left( \frac{\partial v_j}{\partial x_i} + \frac{\partial v_i}{\partial x_j} \right) + B \left( \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} \right) \delta_{ij}$$

- For an elementary flow as the one in fig 1.1-1 (see p.25)  $A = -\mu$  and  $B = \frac{2}{3}\mu - \kappa$ , where  $\kappa$  is known as the dilatational viscosity.  **$\kappa$  is identically zero for monatomic gases at low density.**

- From all that we get the generalization of Newton's Law of Viscosity

$$\tau_{ij} = -\mu \left( \frac{\partial v_j}{\partial x_i} + \frac{\partial v_i}{\partial x_j} \right) + \left( \frac{2}{3}\mu - \kappa \right) \left( \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} \right) \delta_{ij}$$

- We can write the previous equation in vector-tensor notation as follows:

$$\tau = -\mu(\nabla\mathbf{v} + (\nabla\mathbf{v})^t) + \left(\frac{2}{3}\mu - \kappa\right)(\nabla \cdot \mathbf{v})\delta, \text{ in which } \delta \text{ is the unit tensor with components } \delta_{ij}, \nabla\mathbf{v} \text{ is the velocity gradient tensor.}$$

## Section 1.7 - Compressibility of Fluids

- **Bulk Modulus:**  $E_v = -\frac{dp}{dV/V}$ , where  $dp$  is the differential change in pressure needed to create a differential change in volume  $dV$ , of a volume  $V$ . Note that the negative sign is included because an increase in pressure will cause a decrease in volume.

- The bulk modulus is also called the *bulk modulus of elasticity* which has dimensions of pressure ( $FL^{-2}$ ) and it's also expressed as:

$$E_v = \frac{dp}{d\rho/\rho}$$

- Values of  $E_v$  for common liquids are large, for example, at atmospheric pressure and at temperature of 60 °F it would required a pressure of 3120 psi (pound per square inch) to compress a unit of volume of water 1%. Since such large pressures are required to affect a change in volume, it can be concluded that liquids can be considered as *incompressible* for most practical engineering applications.

- **Compression and Expansion of Gases:** When gases are compressed or expanded the relationship between pressure and density depends on the nature of the process. If the process is isothermal then:

$$\frac{p}{\rho} = constant$$

If the compression or expansion is frictionless and no heat is exchanged with the surroundings, then:

$$\frac{p}{\rho^k} = constant, \text{ where } k = C_p/C_v.$$

- ✓ **For an isentropic process:**  $E_v = kp$

- **Speed of Sound:**

$$c = \sqrt{\frac{dp}{d\rho}} = \sqrt{\frac{E_v}{\rho}}$$

### **Section 1.8 - Vapor Pressure**

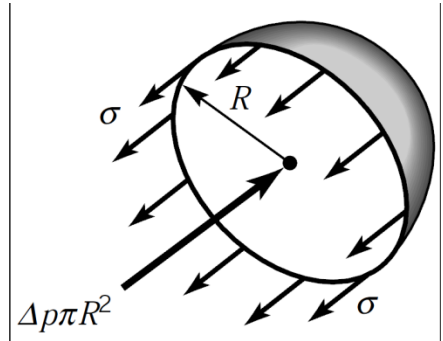
- The **vapor pressure** ( $p_v$ ) is the pressure that the vapor exerts on the liquid surface when an equilibrium condition is reached so that the number of molecules leaving the surface is equal to the number of molecules entering.
- **Boiling**, which is the formation of vapor bubbles within a fluid mass, is initiated **when the absolute pressure in the fluid reaches the vapor pressure**. Boiling can be induced at a given pressure acting on the fluid by raising the temperature, or at a given fluid temperature by lowering the pressure. “To induce boiling >>  $\uparrow T$  or  $\downarrow P$ ”

### Section 1.9 - Surface Tension

- At the interface between a liquid and a gas (or between 2 immiscible liquids) forces develop in the liquid surface which cause the surface to behave as if it were a “skin” or a “membrane” stretched over the fluid mass.
- **Surface Tension** ( $\sigma$ ) can be defined as the intensity of the molecular attraction per unit length along any line in the surface. This quantity has dimensions of  $FL^{-1}$  and it depends on temperature; the surface tension decreases as the temperature increases.
- **Important note:** “Capillary action in small tubes, which involves a liquid, gas & solid interface, is caused by the surface tension.”
- If a small open tube is inserted into water, then the water level in the tube will rise above the water level outside the tube. This height above the water level outside can be obtained by the expression:

$$h = \frac{2\sigma \cos \theta}{\gamma R}$$

Where  $\theta$  is the *angle of contact* between the fluid and the tube  $\gamma$  is the specific weight of the liquid and R is the radius of the tube.



## Suggested Problems

- 1.2** Determine the dimensions in both the FLT and MLT systems, for (a) product of force times volume (b) the product of pressure times mass divided by area and (c) moment of a force divided by velocity.

### Solution:

A) Force\*Volume=  $\boxed{FL^3}$

Force =  $MLT^{-2}$

Force\*Volume=  $\boxed{ML^4/T^2}$

B) (Pressure\*mass)/ Area=

$\boxed{F^2T^2/L^5}$

Pressure\*mass)/ Area=

$\boxed{M^2/(L^3T^2)}$

C) Moment of a force/

Velocity=  $\boxed{FT}$

Moment of a force/

Velocity=  $\boxed{ML/T}$

- 1.8** The volume flow rate,  $Q$ , through a pipe containing a slowly moving liquid is given by the equation

$$Q = \frac{\pi R^4 \Delta P}{8\mu l}$$

where  $R$  is the pipe radius,  $P$  the pressure drop along the fluid,  $\mu$ , a fluid property called viscosity and  $l$  the length of the pipe. What are the dimensions of the constant  $\pi/8$ ?

Would you classify the equation as a general homogenous?

### Solution:

$$\frac{L^3}{T} = \frac{\pi}{8} \left( \frac{L^4 (FL^{-2})}{(FL^{-2}T)L} \right)$$

$$\frac{L^3}{T} = \frac{\pi}{8} \left( \frac{L^3}{T} \right)$$

$\therefore$  The constant ( $\pi/8$ ) is dimensionless, and the equation is a general homogenous equation because it's valid in any consistent unit system.

**1.24** The Specific gravity of mercury at 80°C is 13.4. Determine its density and specific weight.

**Solution:**

$$SG = \frac{\rho}{\rho_{H_2O @ 4^\circ C}} \quad \rho_{H_2O @ 4^\circ C} = 1000 \text{ kg/m}^3 \quad \gamma = \rho g$$

SI Units:

$$\rho_{Hg} = 13.4 \times 1000 \frac{\text{kg}}{\text{m}^3} = \boxed{13,400 \text{ kg/m}^3}$$

$$\gamma = 13,400 \frac{\text{kg}}{\text{m}^3} \times 9.81 \frac{\text{m}}{\text{s}^2} = \boxed{131 \text{ kN}}$$

BG Units:

$$\rho_{Hg} = \left(13.4 \times 10^3 \frac{\text{kg}}{\text{m}^3}\right) \times \left(\frac{6.852 \times 10^{-2} \text{ slug}}{1 \text{ kg}}\right) \times \left(\frac{0.3048 \text{ m}}{1 \text{ ft}}\right)^3 = \boxed{26.0 \text{ slugs/ft}^3}$$

$$\gamma = 26.0 \frac{\text{slugs}}{\text{ft}^3} \times 32.2 \frac{\text{ft}}{\text{s}^2} = \boxed{837 \text{ lb}_f/\text{ft}^3}$$

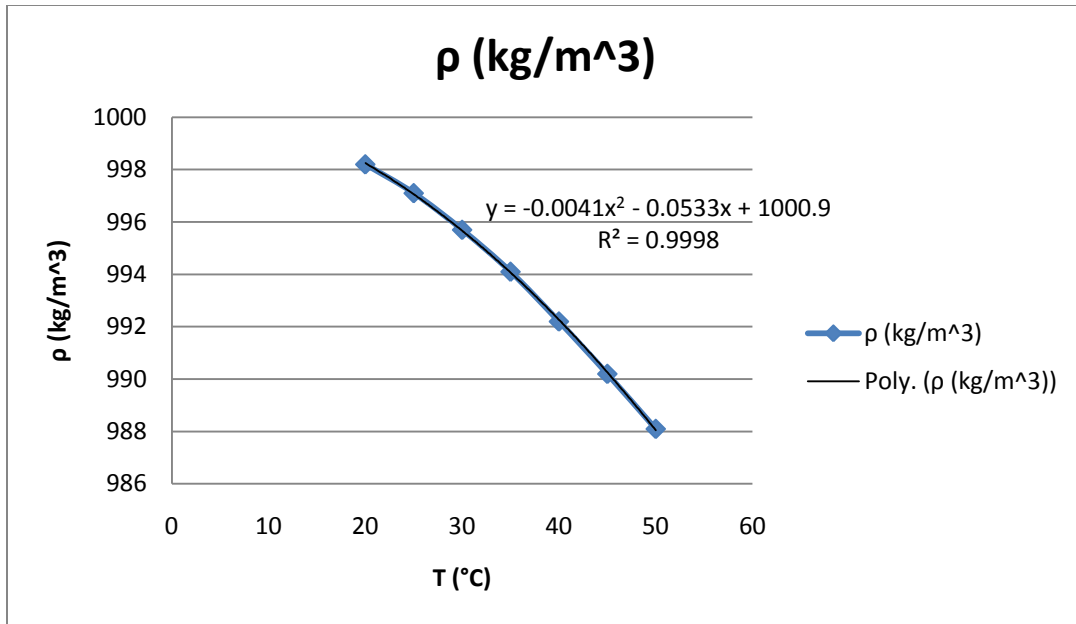
**1.30** The variation in the density of water,  $\rho$ , with the temperature,  $T$ , in the range  $20^\circ\text{C} \leq T \leq 50^\circ\text{C}$ , is given by the following table:

$\rho$ (kg/m <sup>3</sup> )	998.2	997.1	995.7	994.1	992.2	990.2	988.1
T (°C)	20	25	30	35	40	45	50

Use these data to determine an empirical equation of the form  $\rho = C_1 + C_2T + C_3T^2$  which can be used to predict the density over the range indicated. Compare the predicted values with the data given. What is the density of water at 42.1 °C?

**Solution:**

In this exercise we realize a graph of  $\rho$  vs.  $T$  (density versus temperature) and then realize a quadratic fit to obtain the equation of the form  $\rho = C_1 + C_2T + C_3T^2$ .



We obtain that the empirical equation is:  $\rho = -0.0041 T^2 - 0.0533 T + 1001$

So, finally we can calculate the density of water at 42.1 °C:

$$\rho = -0.0041*(42.1)^2 - 0.0533*(42.1) + 1001 = 991.5 \text{ kg/m}^3$$

T (°C)	$\rho$ (kg/m <sup>3</sup> )	$P_{\text{predicted}}$ (kg/m <sup>3</sup> )
20	998.2	998.3
25	997.1	997.1
30	995.7	995.7
35	994.1	994.1
40	992.1	992.3
45	990.2	990.3
50	988.1	988.1

As the table show the prediction was a good one.

**1.42** The viscosity of a soft drink was determined by using a capillary tube viscometer. For this device  $\nu = Kt$  (kinematic viscosity is proportional to time). By what percent is the absolute viscosity ( $\mu$ ) of regular pop greater than that of diet pop?

	Regular Pop	Diet Pop
t (s)	377.8	300.3
SG	1.044	1.003

**Solution:**

$$\nu = \frac{\mu}{\rho} \quad \therefore \quad \mu = \nu\rho \quad \%_{greater} = \left( \frac{\mu_{regular} - \mu_{diet}}{\mu_{diet}} \right) \times 100$$

$$\%_{greater} = \left( \frac{(\nu\rho)_{regular}}{(\nu\rho)_{diet}} - 1 \right) \times 100 = \left( \frac{K(t\rho)_{regular}}{K(t\rho)_{diet}} - 1 \right) \times 100$$

$$\%_{greater} = \left( \frac{(377.8 \times 1.044)}{(300.3 \times 1.003)} - 1 \right) \times 100 = \boxed{30.95 \%}$$

**1.48** Calculate the Reynolds numbers for the flow of water and for air through a 4 mm diameter tube, if the mean velocity is 3 m/s and the temperature is 30 °C in both cases. Assume the air is at standard atmospheric pressure.

$$Re = \frac{\rho V D}{\mu}$$

$\rho$  = density;  $V$  = mean fluid velocity;  $D$  = pipe diameter;  $\mu$  = fluid viscosity

**Solution:**

For H<sub>2</sub>O @ 30 °C (from Appendix B):

$$\rho_w = 995.7 \text{ kg/m}^3$$

$$\mu = 7.975 \times 10^{-4} \text{ N}\cdot\text{s/m}^2$$

For Air @ 30 °C (from Appendix B):

$$\rho_{air} = 1.165 \text{ kg/m}^3$$

$$\mu = 1.86 \times 10^{-5} \text{ N}\cdot\text{s/m}^2$$

$$Re = \frac{(995.7 \frac{kg}{m^3}) \times (3 \frac{m}{s}) \times (4 \times 10^{-3} m)}{7.975 \times 10^{-4} \frac{N \cdot s}{m}} = \boxed{15,000} \quad (\text{for water})$$

$$Re = \frac{(1.165 \frac{kg}{m^3}) \times (3 \frac{m}{s}) \times (4 \times 10^{-3} m)}{1.86 \times 10^{-5} \frac{N \cdot s}{m}} = \boxed{752} \quad (\text{for air})$$

**1.54** Let two layers of fluid are dragged along by the motion of an upper plate as shown in **Fig. P1.54**. The bottom plate is stationary. The top fluid puts a shear stress on the upper plate, and the lower fluid puts a shear stress on the bottom plate. Determine the ratio of these two shear stresses.

**Solution:**

As Fig.P1.54 shows for fluid 1:  $\mu_1 = 0.4 \text{ N}\cdot\text{s}/\text{m}^2$ ,  $u_1 = 3 \text{ m/s}$ ,  $dy = 0.02 \text{ m}$

for fluid 2 :  $\mu_2 = 0.2 \text{ N}\cdot\text{s}/\text{m}^2$ ,  $u_2 = 2 \text{ m/s}$ ,  $dy = 0.02 \text{ m}$

$$\tau_1 = \mu_1 \left( \frac{du}{dy} \right)_{top} = \left( 0.4 \text{ N} \cdot \frac{\text{s}}{\text{m}} \right) \times \left( \frac{3 \frac{\text{m}}{\text{s}} - 2 \frac{\text{m}}{\text{s}}}{0.02 \text{ m}} \right) = 20 \text{ N/m}^2$$

$$\tau_2 = \mu_2 \left( \frac{du}{dy} \right)_{bottom} = \left( 0.2 \text{ N} \cdot \frac{\text{s}}{\text{m}} \right) \times \left( \frac{2 \frac{\text{m}}{\text{s}} - 0 \frac{\text{m}}{\text{s}}}{0.02 \text{ m}} \right) = 20 \text{ N/m}^2$$

$$\boxed{\text{ratio} = \frac{\tau_1}{\tau_2} = 1}$$

**1.58** A Newtonian fluid with  $SG = 0.92$  &  $\nu = 4 \times 10^{-4} \text{ m}^2/\text{s}$  past a fixed surface. Due to the no-slip condition, the velocity at the fixed surface is zero, and the velocity profile near the surface is shown in **Fig. P1.58**. Determine the magnitude and direction of the shearing stress developed on the plate. Express your answer in terms of  $U$  and  $\delta$ , with  $U$  and  $\delta$  expressed in units of meters per second and meter, respectively.

$$\frac{u}{U} = \frac{3y}{2\delta} - \frac{1}{2} \left( \frac{y}{\delta} \right)^3$$

**Solution:**

$$u = U \left[ \frac{3y}{2\delta} - \frac{1}{2} \left( \frac{y}{\delta} \right)^3 \right] \quad \frac{du}{dy} = U \left[ \frac{3}{2\delta} - \frac{3y^2}{2\delta^3} \right]$$

$$\tau_{surface} = \mu \left( \frac{du}{dy} \right)_{y=0} \quad @ y = 0 \Rightarrow \frac{du}{dy} = U \left[ \frac{3}{2\delta} \right]$$

$$\mu = \nu \rho \Rightarrow \tau_{surface} = \nu \rho \frac{3U}{2\delta}$$

$$\tau_{surface} = \left( 4 \times 10^{-4} \frac{\text{m}^2}{\text{s}} \right) \left( 920 \frac{\text{kg}}{\text{m}^3} \right) \left( \frac{3}{2} \right) \frac{U}{\delta}$$

$$\boxed{\tau_{surface} = 0.552 \frac{U}{\delta} \text{ N/m}^2, \text{ acting to the left on plate}}$$

## Conversion Factors:

**TABLE 1.3**  
**Conversion Factors from BG and EE Units to SI Units<sup>3</sup>**

	To Convert from	to	Multiply by
Acceleration	ft/s <sup>2</sup>	m/s <sup>2</sup>	3.048 E - 1
Area	ft <sup>2</sup>	m <sup>2</sup>	9.290 E - 2
Density	lbm/ft <sup>3</sup>	kg/m <sup>3</sup>	1.602 E + 1
	slugs/ft <sup>3</sup>	kg/m <sup>3</sup>	5.154 E + 2
Energy	Btu	J	1.055 E + 3
	ft · lb	J	1.356
Force	lb	N	4.448
Length	ft	m	3.048 E - 1
	in.	m	2.540 E - 2
	mile	m	1.609 E + 3
Mass	lbm	kg	4.536 E - 1
	slug	kg	1.459 E + 1
Power	ft · lb/s	W	1.356
	hp	W	7.457 E + 2
Pressure	in. Hg (60 °F)	N/m <sup>2</sup>	3.377 E + 3
	lb/ft <sup>2</sup> (psf)	N/m <sup>2</sup>	4.788 E + 1
	lb/in. <sup>2</sup> (psi)	N/m <sup>2</sup>	6.895 E + 3
Specific weight	lb/ft <sup>3</sup>	N/m <sup>3</sup>	1.571 E + 2
Temperature	°F	°C	$T_C = (5/9)(T_F - 32°)$
	°R	K	5.556 E - 1
Velocity	ft/s	m/s	3.048 E - 1
	mi/hr (mph)	m/s	4.470 E - 1
Viscosity (dynamic)	lb · s/ft <sup>2</sup>	N · s/m <sup>2</sup>	4.788 E + 1
Viscosity (kinematic)	ft <sup>2</sup> /s	m <sup>2</sup> /s	9.290 E - 2
Volume flowrate	ft <sup>3</sup> /s	m <sup>3</sup> /s	2.832 E - 2
	gal/min (gpm)	m <sup>3</sup> /s	6.309 E - 5

<sup>3</sup>If more than four-place accuracy is desired, refer to Appendix E.

TABLE 1.4

Conversion Factors from SI Units to BG and EE Units<sup>a</sup>

	To Convert from	to	Multiply by
Acceleration	m/s <sup>2</sup>	ft/s <sup>2</sup>	3.281
Area	m <sup>2</sup>	ft <sup>2</sup>	1.076 E + 1
Density	kg/m <sup>3</sup>	lbm/ft <sup>3</sup>	6.243 E - 2
	kg/m <sup>3</sup>	slugs/ft <sup>3</sup>	1.940 E - 3
Energy	J	Btu	9.478 E - 4
	J	ft · lb	7.376 E - 1
Force	N	lb	2.248 E - 1
Length	m	ft	3.281
	m	in.	3.937 E + 1
	m	mile	6.214 E - 4
Mass	kg	lbm	2.205
	kg	slug	6.852 E - 2
Power	W	ft · lb/s	7.376 E - 1
	W	hp	1.341 E - 3
Pressure	N/m <sup>2</sup>	in. Hg (60 °F)	2.961 E - 4
	N/m <sup>2</sup>	lb/ft <sup>2</sup> (psf)	2.089 E - 2
	N/m <sup>2</sup>	lb/in. <sup>2</sup> (psi)	1.450 E - 4
Specific weight	N/m <sup>3</sup>	lb/ft <sup>3</sup>	6.366 E - 3
Temperature	°C	°F	$T_F = 1.8 T_C + 32^\circ$
	K	°R	1.800
Velocity	m/s	ft/s	3.281
	m/s	mi/hr (mph)	2.237
Viscosity (dynamic)	N · s/m <sup>2</sup>	lb · s/ft <sup>2</sup>	2.089 E - 2
Viscosity (kinematic)	m <sup>2</sup> /s	ft <sup>2</sup> /s	1.076 E + 1
Volume flowrate	m <sup>3</sup> /s	ft <sup>3</sup> /s	3.531 E + 1
	m <sup>3</sup> /s	gal/min (gpm)	1.585 E + 4

<sup>a</sup>If more than four-place accuracy is desired, refer to Appendix E.

✓ Here are some links of interesting videos about Fluids Mechanics:

Experiment viscosity: <http://www.youtube.com/watch?v=oHxNnn>

Gas expansion: <http://www.youtube.com/watch?v=dQeCEqkE9eE>

Surface tension: [http://www.youtube.com/watch?v=HQ8FP0sa\\_hk](http://www.youtube.com/watch?v=HQ8FP0sa_hk)

Capillary rise: <http://www.youtube.com/watch?v=6zmG2ksqSIc>

## References:

B.R. Munson, D.F. Young and T.H. Okiishi, *Fundamentals of Fluid Mechanics*, 5<sup>th</sup> edition, New York, N.Y.: John Wiley & Sons, Inc. 2002

Wikipedia, *Shear Stress*, [http://en.wikipedia.org/wiki/Shear\\_stress](http://en.wikipedia.org/wiki/Shear_stress), Wikipedia the Free Encyclopedia, 2009.

R. Byron, W. Stewart, E. Lightfoot *Transport Phenomena*, 2<sup>nd</sup> edition, New York, N.Y.: John Wiley & Sons, Inc. 2002