

Chapter 4: Kinematics of fluid motion
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4.1 The Velocity Field

In general the fluid flow is a net motion of molecules from one point to another as function of time.

For study fluid motion we employ the continuum hypothesis and consider fluids to be made up of fluid particles. Each particle contains numerous molecules. Thus, we can describe the fluid motion in terms of velocity and acceleration of fluid particles.

At a given instant in time a description of any fluid property may be given as a function of fluid's location, **field representation**.

One of the most important fluid variables is the **velocity field**,

$$\mathbf{V} = u(x,y,z,t)\mathbf{i} + v(x,y,z,t)\mathbf{j} + w(x,y,z,t)\mathbf{k}$$

The velocity of a particle is the time rate of change of the position vector for that particle.

$$\mathbf{V} = d\mathbf{r} / dt$$

$$\mathbf{V} = \mathbf{V}(x,y,z,t)$$

$$V = |\mathbf{V}| = (u^2 + v^2 + w^2)^{1/2}$$

Since the velocity is a vector, it has a direction and a magnitude. The magnitude of \mathbf{V} is the speed of the fluid. A change in velocity results in an acceleration. This acceleration may be due to a change in speed and/or direction.

4.1.1 Eulerian and Lagrangian Flow Descriptions

Are two general approaches in analyzing fluid mechanics problems. The first method, called **Eulerian method**, used the field concept. From this method we obtain information about the flow in terms of what happens at fixed points in space as the fluid flows past those points. The second method, called **Lagrangian method**, involves following individual fluid particles as they move about and determining how the fluid properties associated with these particles change as a function of time.

Example: smoke discharging from a chimney. In the Eulerian method one may attach a temperature-measuring device to the top of the chimney and record the temperature at that point as a function of time. In Lagrangian method one would attach temperature-measuring device to a particular fluid particle and record that particle's temperature as it moves about.

In fluid mechanics it is usually easier to use Eulerian method in experimental and analytical investigations. There are certain instances in which the Lagrangian method is more convenient.

4.1.2 One-, Two-, and Three-Dimensional Flows

Generally, a fluid flow is a rather complex three-dimensional, time-dependent phenomenon. Neglect of one or two of the velocity components would lead to considerable misrepresentation of the effects produced by the actual flow. In many situations one of the velocity components may be small relative to the two other components and may be reasonable assume two-dimensional flow. For simplify a flow analysis assuming those two velocity components are negligible and approximated a one-dimensional flow field. There are also many flow situations for which use one-dimensional flow field assumption will give completely erroneous results.

$$\mathbf{V} = \mathbf{V}(x,y,z,t) = u\mathbf{i} + v\mathbf{j} + w\mathbf{k}$$

$$\mathbf{V} = \mathbf{V}(x,y,t) = u\mathbf{i} + v\mathbf{j}$$

$$\mathbf{V} = \mathbf{V}(x,t) = u\mathbf{i}$$

4.1.3 Steady and Unsteady Flows

Steady flow (laminar flow)

- the values of all fluid properties (velocity, temperature, density, etc.) at any fixed point are independent of time.
- the velocity at a given point in space does not vary with time $\frac{\partial v}{\partial t} = 0$

Unsteady Flow (turbulent flow)

- the velocity does vary with time.
- unsteady flow are usually more difficult to analyze and to investigate experimentally than are steady flows.
- types of unsteady flows are nonperiodic flow, periodic flow and truly random flow.
- Example of **nonperiodic** unsteady flow is that produced by turning off a faucet to stop the flow of water.
- Example of **periodic** unsteady effect, the periodic injection of the air-gasoline mixture into the cylinder of an automobile engine.
- Example of **truly random** flow, the turbulent flow observed in the “irregular” splashing of water from a faucet onto the sink below it and the “irregular” gustiness of the wind represents another random turbulent flow.

4.1.4 Streamlines, Streaklines, and Pathlines

The purpose of this section is discussing the use of various methods such as streamlines, streaklines, and pathlines for visualizing and describing flows.

- **Streamlines** are lines tangent to the velocity flow. This line often used in analytical work. Examples of this are if the flow is steady, nothing at a fixed point (including the velocity direction) changes with time, so the streamlines are fixed lines in space. For unsteady flows the streamlines may change shape with time.

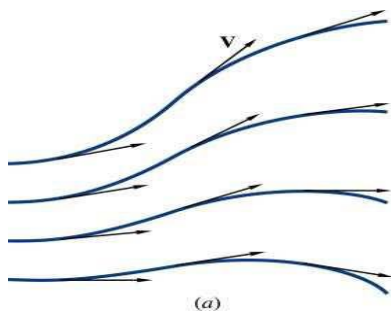
Streamlines are obtained analytically by integrating the equations defining lines tangent to the velocity field. For two-dimensional flows the slope of the streamline, must be equal to the tangent of the angle that the velocity vector makes with the x axis

or
$$\frac{\partial y}{\partial x} = \frac{v}{u} \quad (4.1)$$

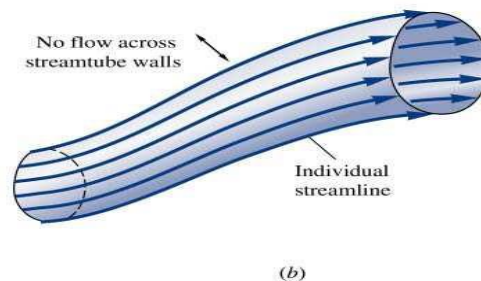
If the velocity field is known as a function of x and y (and t if the flow is unsteady), this equation can be integrated to give the equation of the streamlines. For unsteady flow there is no easy way to produce streamlines experimentally in the laboratory.

For practice study example 4.2

- A ***streakline*** particles in a flow that have previously passed through a common point. Streaklines are more of a laboratory tool than an analytical tool in other words streakline is often used in experimental work. This line can be obtained by taking instantaneous photographs of marked particles that all passed through a given location in the flow field at some earlier time. Such a line can be produced by continuously injecting marked fluid (neutrally buoyant smoke in air, or dye in water) at a given location.
- A ***pathline*** locus of points transverse by a given particle as it travels in a field of flow. The pathline is a Lagrangian concept that can be produced in the laboratory by marking a fluid particle (dyeing a small fluid element) and taking a time exposure photograph of its motion.



Streamlines



Streamtubes

Streamtube-tube whose walls are streamlines

For practice study example 4.3

Pathlines, Streamlines, and Streaklines are the same for steady flows. For unsteady flows none of these three types of lines need be the same.

4.2 The Acceleration Field

We can describe fluid motion by Lagrangian or Eulerian description. For Lagrangian method we describe the fluid acceleration as is done in solid body dynamics $\mathbf{a} = \mathbf{a}(t)$. In Eulerian description we describe the **acceleration field** as a function of position and time. In this section discuss how to obtain the acceleration field if the velocity field is known. The acceleration of a particle is the time rate of change of its velocity from one point to another.

4.2.1 The Material Derivative

The particle's velocity, \mathbf{V} , is a function of its location and time.

$$\mathbf{V} = \mathbf{V}(\mathbf{r},t) = \mathbf{V}[x(t), y(t), z(t), t]$$

We use the differentiation to obtain the acceleration of particle

$$\mathbf{a}(t) = \frac{d\mathbf{V}}{dt} = \frac{\partial\mathbf{V}}{\partial t} + \frac{\partial\mathbf{V}}{\partial x} \frac{dx}{dt} + \frac{\partial\mathbf{V}}{\partial y} \frac{dy}{dt} + \frac{\partial\mathbf{V}}{\partial z} \frac{dz}{dt}$$

Using the fact that the velocity component are: $u = dx/dt$, $v = dy/dt$, $w = dz/dt$

$$\mathbf{a}(t) = \frac{\partial\mathbf{V}}{\partial t} + u \frac{\partial\mathbf{V}}{\partial x} + v \frac{\partial\mathbf{V}}{\partial y} + w \frac{\partial\mathbf{V}}{\partial z}$$

Component of the acceleration

$$a_x = \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z}$$

$$a_y = \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z}$$

$$a_z = \frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z}$$

Shorthand notation

$$\mathbf{a} = \frac{D\mathbf{V}}{Dt}$$

The material derivative or substantial derivative. This concept is very useful in analysis involving various fluid parameters, not just the acceleration.

$$\frac{D(\)}{Dt} = \frac{\partial(\)}{\partial t} + u \frac{\partial(\)}{\partial x} + v \frac{\partial(\)}{\partial y} + w \frac{\partial(\)}{\partial z}$$

$$\frac{D(\)}{Dt} = \frac{\partial(\)}{\partial t} + (\mathbf{V} \cdot \nabla)(\)$$

4.2.2 Unsteady Effects

The material derivative formula contains two types of terms: time and spatial derivative. The time derivatives are denoted as the local derivative. They represent effects of the unsteadiness of the flow. For steady flow the time derivative is zero throughout the flow field $\partial(\)/\partial t = 0$ and the local effect vanished. If flow is unsteady, its parameter values (velocity, temperature, density, etc.) at any location may change with time.

4.2.3 Convective Effects

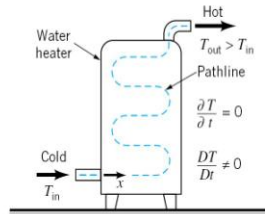
- The convective derivative is a result of the spatial variation of the flow.

$$\frac{D(\)}{Dt} \equiv \frac{\partial(\)}{\partial t} + u \frac{\partial(\)}{\partial x} + v \frac{\partial(\)}{\partial y} + w \frac{\partial(\)}{\partial z}$$

That is termed the material derivative or substantial derivative, the portion of the material derivative represented by the spatial derivatives, is termed the convective derivative.

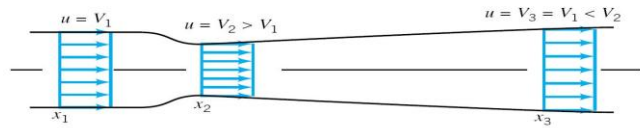
$$\frac{D(\)}{Dt} = \frac{\partial(\)}{\partial t} + (\mathbf{V} \cdot \nabla)(\)$$

- It represents the fact that a flow property associated with a fluid particle may vary because of the motion of the particle from one point in the space where the parameter has one value to another point in space where its value is different.
- This contribution can occur whether the flow is steady or unsteady.



■ FIGURE 4.6 Steady-state operation of a water heater.

- The temperature of a particle changes as it flows through a water heater. The water entering has the same cold temperature, and the water leaving has the same hot temperature. The flow is steady, and the temperature of each particle increases as it passes through the heater.
- The term $Dt/Dt \neq 0$ because of the convective term in the total derivative of temperature.
- A fluid particle traveling along a non-constant temperature path ($dT/dx \neq 0$) at a specific speed u , will have its temperature change with time at a rate of $DT/Dt = u dT/dx$, even though the flow is steady ($dT/dt = 0$).

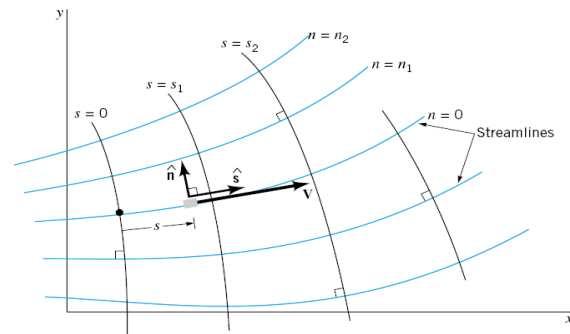


■ FIGURE 4.7 Uniform, steady flow in a variable area pipe.

- The assumptions are that the flow is steady and one dimensional with velocity that increases and decreases in the flow direction. As it flows to section 1 to 2, the velocity increases from V_1 to V_2 . As $dV/dt = 0$, fluid particles experience an acceleration given by $a_x = udu/dx$. For $x_1 < x < x_2$, it is seen that $du/dx > 0$ so $a_x > 0$, the fluid accelerates, and for opposite if $x_2 < x < x_3$, now it is seen that $du/dx < 0$, so $a_x < 0$, the fluid decelerate.

4.2.4 Streamline Coordinates

- Streamline coordinates provide a natural coordinate system for a flow.
- It is convenient to use a coordinate system define in terms of streamlines of the flow.



■ FIGURE 4.8 Streamline coordinate system for two-dimensional flow.

- This is an example of steady, two dimensional flows, that the flow can described either in terms of the usual x, y coordinate system, or the streamline coordinate system.
- In the streamline system the flow is describe in terms of one coordinate along the streamlines, denoted s , and the second coordinate normal (perpendicular) to the streamline (denoted n), the unit vector of this two directions are denoted as \hat{s} and \hat{n} .
- It's important that do not confuse the coordinate system s that is a scalar, with the unit vector along the streamline direction \hat{s} .
- The lines of constant s and n are not necessarily straight, and at any point of s and n directions are perpendicular.
- One of the major advantages of using the streamline coordinate system is that the velocity is always tangent to the s direction, $\mathbf{V} = v \hat{s}$. These allow simplifications in describing acceleration and solve equations.

- The acceleration can be determined by $\mathbf{a} = \mathbf{D}\mathbf{V}/\mathbf{D}t = a_s \hat{\mathbf{s}} + a_n \hat{\mathbf{n}}$.

Streamline and normal components of acceleration

- Use the material derivative because by definition the acceleration is the time rate of change of the velocity of a particle as it moves about.
- Use the fact for steady flow nothing changes with time at a point for simplified the acceleration. Stream line and normal components off acceleration occur even in steady flow.
- The figure 4.9 (Pag. 167) it seen that the magnitude of $d\hat{\mathbf{s}}/ds$ is equal to the inverse of the radius of $\hat{\mathbf{s}}$ curvature of the streamline, R at a point.
- The acceleration for steady, two dimensional flows can be written in terms of its stream wise and normal components.

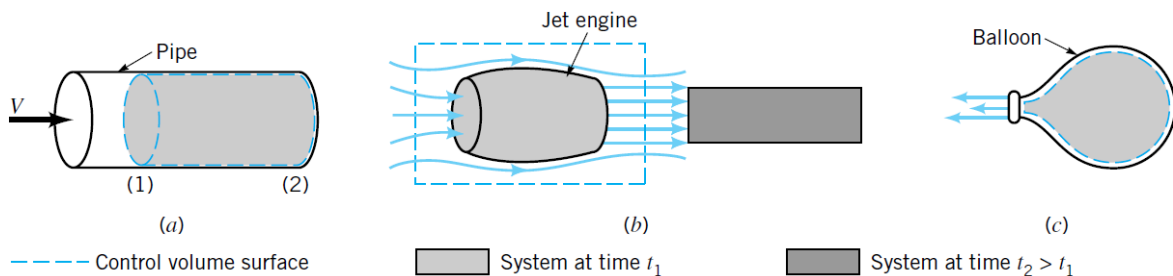
$$\mathbf{a} = V \frac{\partial V}{\partial s} \hat{\mathbf{s}} + \frac{V^2}{\mathcal{R}} \hat{\mathbf{n}} \quad \text{or} \quad a_s = V \frac{\partial V}{\partial s}, \quad a_n = \frac{V^2}{\mathcal{R}}$$

Convective

Centrifugal acceleration

4.3 Control Volume and System Representation

- There are various ways that these governing laws can be applied to a fluid, including the system approach and the control volume approach.
- System** is a collection of matter of fixed identity, which may move, flow, and interacts with its surroundings.
- Control volume**, on the other hand, is a volume in space through which fluid may flow.
- The system may interact with its surroundings by various means. It may continually change size and shape, but it always contains the same mass.
- In fluid mechanics, it is often quite difficult to identify and keep track of a specific quantity of matter. A finite portion of a fluid contains an uncountable number of fluid particles that move about quite freely, unlike a solid that may deform but usually remains relatively easy to identify.



- The matter within a control volume may change with time as the fluid flows through it. Similarly, the amount of mass within the volume may change with time. The control volume itself is a specific geometric entity, independent of the flowing fluid.
- Examples of control volumes and *control surfaces* (the surface of the control volume) are shown in Fig. 4.10.
 1. For case (a), fluid flows through a pipe.
 2. Control volume is the rectangular volume surrounding the jet engine shown in Fig.4.10b.
 3. The deflating balloon shown in Fig. 4.10c provides an example of a deforming control volume.
- The majority of the problems we will analyze can be solved by using a fixed, nondeforming control volume. In some instances, however, it will be advantageous, in fact necessary, to use a moving, deforming control volume.
- All of the laws governing the motion of a fluid are stated in their basic form in terms of a system approach. For example, “the mass of a system remains constant,” or “the time rate of change of momentum of a system is equal to the sum of all the forces acting on the system.”

4.4 The Reynolds Transport Theorem

The Reynolds transport theorem provides the tool to describe the laws governing fluids motion using both system concepts and control volume concepts. All physical laws are stated in terms of various physical parameters. Velocity, acceleration, mass, temperature, and momentum are but a few of the more common parameters. Let B represent any of these fluid parameters and b represent the amount of that parameter per unit mass.

$$B = mb$$

Thus, if $\mathbf{B} = m\mathbf{V}$ the momentum of the mass, then $\mathbf{b} = \mathbf{V}$. (*The momentum per unit mass is the velocity.*) The parameter B is termed an *extensive property* and the parameter b is termed an *intensive property*. The value of B is directly proportional to the amount of the mass being considered, whereas the value of b is independent of the amount of mass. The amount of an extensive property that a system possesses at a given instant, can be determined by adding up the amount associated with each fluid particle in the system.

$$B_{sys} = \int \rho b dV$$

Most of the laws governing fluid motion involve the time rate of change of an extensive property of a fluid system—the rate at which the momentum of a system changes with time, the rate at which the mass of a system changes with time, and so on. Thus, we often encounter terms such as

$$\frac{dB_{sys}}{dt} = \frac{d(\int \rho b dV)}{dt}$$

To formulate the law into a control volume approach, we can use the previous equation in term of control volume, B_{cv}

$$\frac{dB_{cv}}{dt} = \frac{d(\int \rho b dV)}{dt}$$

We will learn that even for those instances when the control volume and the system momentarily occupy the same volume in space, the two quantities $\frac{dB_{sys}}{dt}$ and $\frac{dB_{cv}}{dt}$ need not be the same. The

Reynolds transport theorem provides the relationship between the time rate of change of an extensive property for a system and that for a control volume.

4.4.1 Derivation of the Reynolds Transport Theorem

The Reynolds transport theorem relating system concepts to control volume concepts can be obtained easily for the one-dimensional flow through a fixed control volume as is shown in Fig. 4.11a. We consider the control volume to be that stationary volume within the pipe or duct between sections (1) and (2) as indicated. The system that we consider is that fluid occupying the control volume at some initial time t . A short time later, at time $t + \delta t$, the system has moved slightly to the right. The fluid particles that coincided with section (2) of the control surface at time t have moved a distance $\delta l_2 = V_2 \delta t$ to the right, where V_2 is the velocity of the fluid as it passes section (2). Similarly, the fluid initially at section (1) has moved a distance $\delta l_1 = V_1 \delta t$ where V_1 is the fluid velocity at section (1). We assume the fluid flows across sections (1) and (2) in a direction normal to these surfaces and that V_1 and V_2 are constant across sections (1) and (2).

If B is an extensive parameter of the system, then the value of it for the system at time t is

$$B_{sys}(t) = B_{cv}(t)$$

Its value at $t + \delta t$ time and the change in the amount of B in the system in the time interval δt divided by this time

$$\frac{\delta B_{sys}}{\delta t} = \frac{B_{cv}(t + \delta t) - B_{cv}(t)}{\delta t} - \frac{B_{i}(t + \delta t)}{\delta t} + \frac{B_{i}(t)}{\delta t}$$

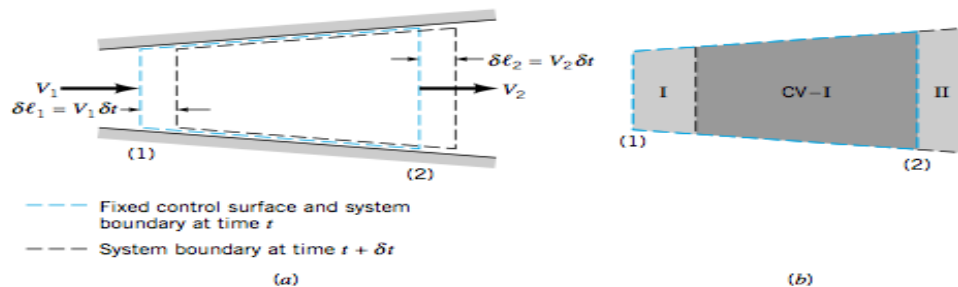


FIGURE 4.11 Control volume and system for flow through a variable area pipe.

In the limit $\delta t \rightarrow 0$ of the first term in the previous equation seen to be the time rate of change of the amount of B within the control volume

$$\lim_{\delta t \rightarrow 0} \frac{B_{cv}(t + \delta t) - B_{cv}(t)}{\delta t} = \frac{\partial B_{cv}}{\partial t} = \frac{\partial (\int \rho b dV)}{\partial t}$$

The third term its convert,

$$\dot{B}_{out} = \lim_{\delta t \rightarrow 0} \frac{B_{i}(t + \delta t)}{\delta t} = \rho_2 A_2 V_2 b_2$$

The equation of inflow \dot{B}_{in} it's similar to a previous equation but in term $\rho_1 A_1 V_1 b_1$.

The resultant equation we see that the relationship between the time rate of change of B for the system and that for the control volume is given by

$$\frac{DB_{sys}}{Dt} = \frac{\partial B_{cv}}{\partial t} + \dot{B}_{out} - \dot{B}_{in}$$

The other forms to explain the Reynolds Transport Theorem it presents in the other page.

Reynolds Transport Theorem

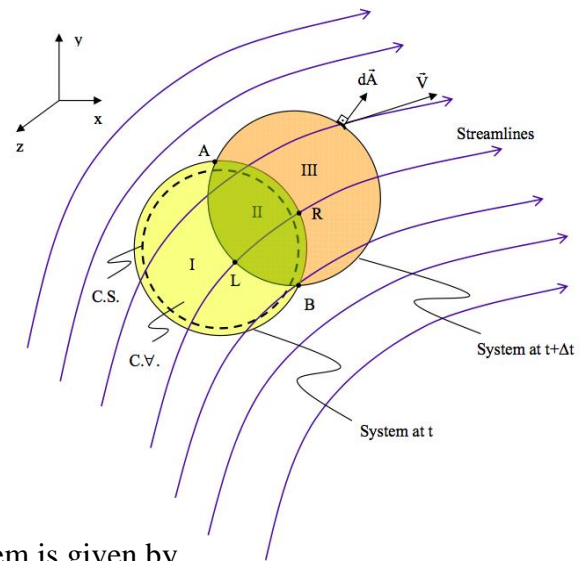
This theorem transforms the system formulation to control volume formulation; which is given by the following expression

$$\frac{DB_{sys}}{Dt} = \frac{\partial}{\partial t} \int_{C.V.} b \rho dV + \int_{C.S.} b \rho (\vec{V} \cdot \vec{n}) dA$$

1. The first integral represents the rate of change of the extensive property in the control volume.
2. The second integral represents the flux of the extensive property across the control surface.

Derivation of Reynolds Transport Theorem

Consider an arbitrary flow field defined by $V(x,y,z,t)$ and a system moving with this flow field. Let the boundary of the C.V. coincide with the boundary of the system at time t . They both occupy regions I and II at time t . As time passes to $t + \Delta t$, the system occupies regions II and III. The system has been chosen so that the mass within the region I enter the C.V. during time interval Δt , and the mass in region III leaves the C.V. during the same time interval.



Mathematically, the rate of change of B for the system is given by

$$\frac{DB_{sys}}{Dt} = \lim_{\Delta t \rightarrow 0} \frac{(B_{sys})_{t+\Delta t} - (B_{sys})_t}{\Delta t} \text{ or}$$

$$\frac{DB_{sys}}{Dt} = \lim_{\Delta t \rightarrow 0} \left[\frac{\left(\int_{V_3} b \rho dV + \int_{V_2} b \rho dV \right)_{t+\Delta t} - \left(\int_{V_1} b \rho dV + \int_{V_2} b \rho dV \right)_t}{\Delta t} \right]$$

Since the limit of the sums equals the sum of the limits one can write,

$$\frac{DB_{\text{sys}}}{Dt} = \lim_{\Delta t \rightarrow 0} \left[\underbrace{\frac{\left(\int_{V_2} b \rho dV \right)_{t+\Delta t} - \left(\int_{V_2} b \rho dV \right)_t}{\Delta t}}_1 + \underbrace{\lim_{\Delta t \rightarrow 0} \frac{\left(\int_{V_3} b \rho dV \right)_{t+\Delta t}}{\Delta t}}_2 - \underbrace{\lim_{\Delta t \rightarrow 0} \frac{\left(\int_{V_1} b \rho dV \right)_t}{\Delta t}}_3 \right]$$

Since the volume V_2 becomes that of the C.V. as $\Delta t \rightarrow 0$ in the limit we have for the 1st limit expression,

$$\lim_{\Delta t \rightarrow 0} \left[\frac{\left(\int_{V_2} b \rho dV \right)_{t+\Delta t} - \left(\int_{V_2} b \rho dV \right)_t}{\Delta t} \right] = \frac{\partial}{\partial t} \int_{\text{C.V.}} b \rho dV$$

In the 2nd limit term the integral $\int b \rho dV$ represents the amount of property B that has V_3 crossed the part of C.S. (say ARB). This value divided by Δt gives the average rate of flux of B across ARB during the time interval Δt . Then the limit of that term can be written as

$$\lim_{\Delta t \rightarrow 0} \frac{\left(B_{\text{III}} \right)_{t+\Delta t}}{\Delta t} = \lim_{\Delta t \rightarrow 0} \frac{\left(\int_{V_3} b \rho dV \right)_{t+\Delta t}}{\Delta t} = \int_{\text{C.S.III}} b \rho (\vec{V} \cdot \vec{n}) dA$$

Similarly the last term represents the exact rate of influx of B into the C.V. at time t through the part of C.S. (ALB). This may be written as

$$\lim_{\Delta t \rightarrow 0} \frac{\left(\int_{V_1} b \rho dV \right)_t}{\Delta t} = \int_{\text{C.S.I}} b \rho (\vec{V} \cdot \vec{n}) dA$$

The last two limiting processes represent the net flux rate of B across the entire control surface. They can be replaced by one term as follows, where \vec{V} is the velocity vector and \vec{n} is the unit vector pointing outward from an enclosed region.

$$\int_{\text{C.S.}} b \rho (\vec{V} \cdot \vec{n}) dA \qquad \frac{DB_{\text{sys}}}{Dt} = \underbrace{\frac{\partial}{\partial t} \int_{\text{C.V.}} b \rho dV}_1 + \underbrace{\int_{\text{C.S.}} b \rho (\vec{V} \cdot \vec{n}) dA}_3$$

Finally we have:

The physical meanings of the terms are:

1. The total rate of change of any arbitrary extensive property B , of the system;
2. The time rate of change of the arbitrary extensive property B within the control volume.
3. The net rate of flux of the extensive property through the control surface.

V is measured in the above equations relative to control volume which is fixed relative to the reference coordinates x , y and z . Thus, the time rate of change of the arbitrary extensive property B within the control volume must be evaluated by an observer fixed in the control volume.

4.4.2 Physical Interpretation

The Reynolds transport theorem is widely used in fluid mechanics. At first it appears to be a rather formidable mathematical expression perhaps one to be steered clear of if possible. However, a physical understanding of the concepts involved will show that it is a rather straightforward, relatively easy-to-use tool. Its purpose is to provide a link between control volume ideas and system ideas.

The left side is the time rate of change of an arbitrary extensive parameter of a system. This may represent the rate of change of mass, momentum, energy, or angular momentum of the system, depending on the choice of the parameter B .

Because the system is moving and the control volume is stationary, the time rate of change of the amount of B within the control volume is not necessarily equal to that of the system. The first term on the right side of equations represents the rate of change of B within the control volume as the fluid flows through it. Recall that b is the amount of B per unit mass, so that is the amount of B in a small volume. Thus, the time derivative of the integral of throughout the control volume is the time rate of change of B within the control volume at a given time.

The last term in this equation represents the net flow rate of the parameter B across the entire control surface. Over a portion of the control surface this property is being carried out of the control volume over other portions it is being carried into the control volume. The mass flow rate through area element given by is positive for outflow $\int_{\text{out}} \rho \mathbf{v} \cdot \mathbf{n} dA$ and negative for inflow $\int_{\text{in}} \rho \mathbf{v} \cdot \mathbf{n} dA$. Each fluid particle or fluid mass carries a certain amount of B with it, as given by the product of B per unit mass, b , and the mass. The rate at which this B is carried across the control surface is given by the area integral term of equation. This net rate across the entire control surface may be negative, zero, or positive depending on the particular situation involved.

4.4.3 Relationship to Material Derivative

The physical interpretation of this derivative is that it provides the time rate of change of a fluid property (temperature, velocity, etc.) associated with a particular fluid particle as it flows. The value of that parameter for that particle may change because of unsteady effects or because of effects associated with the particle's motion.

Careful consideration of Eq. 4.19 indicates the same type of physical interpretation for the Reynolds transport theorem. The term involving the time derivative of the control volume integral represents unsteady effects associated with the fact that values of the parameter within the control volume may change with time. For steady flow this effect vanishes—fluid flows through the control volume but the amount of any property, B , within the control volume is constant in time. The term involving the control surface integral represents the convective effects associated with the flow of the system across the fixed control surface. The sum of these two terms gives the rate of change of the parameter B for the system. This corresponds to the interpretation of the material derivative, in which the sum of the unsteady effect and the convective effect gives the rate of change of a parameter for a fluid particle. As is discussed in **Section 4.2**, the material derivative operator may be applied to scalars or vectors. This is also true for the Reynolds transport theorem. The particular parameters of interest, B and b , may be scalars or vectors.

Thus, both the material derivative and the Reynolds transport theorem equations represent ways to transfer from the Lagrangian to the Eulerian viewpoint. The material derivative Eq. 4.5 is essentially the infinitesimal equivalent of the finite size Reynolds transport theorem.

4.4.4 Steady Effects

Consider a steady flow so that Eq. 4.19 reduces to

$$\frac{DB_{\text{sys}}}{Dt} = \int \rho b \mathbf{V} \cdot \mathbf{\check{n}} dA$$

In such cases if there is to be a change in the amount of B associated with the system, there must be a net difference in the rate that B flows into the control volume compared with the rate that it flows out of the control volume. That is, the integral of over the inflow portions of the control surface would not be equal and opposite to that over the outflow portions of the surface.

Consider steady flow through the “black box” control volume. If the parameter B is the mass of the system, the left-hand side of Eq. 4.20 is zero. Hence, the flow rate of mass into the box must be the same as the flow rate of mass out of the box because the right-hand side of Eq. 4.20 represents the net flow rate through the control surface. On the other hand, assume the parameter B is the momentum of the system. The momentum of the system need not be constant. In fact, according to Newton’s second law the time rate of change of the system momentum equals the net force, \mathbf{F} , acting on the system. In general, the left-hand side of Eq. 4.20 will therefore be nonzero. Thus, the right-hand side, which then represents the net flux of momentum across the control surface, will be nonzero. The flow rate of momentum into the control volume need not be the same as the flux of momentum from the control volume. We will investigate these concepts much more fully in Chapter 5. They are the basic principles describing the operation of such devices as jet or rocket engines.

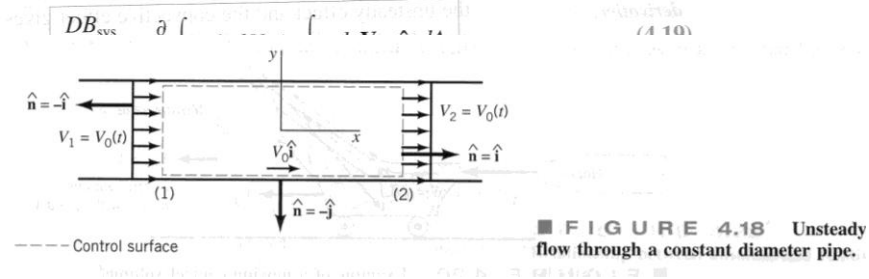
For steady flows the amount of the property B within the control volume does not change with time. The amount of the property associated with the system may or may not change with time, depending on the particular property considered and the flow situation involved. The difference between that associated with the control volume and that associated with the system is determined by the rate at which B is carried across the control surface—the term $\int_{cs} \rho b \mathbf{V} \cdot \mathbf{\check{n}} dA$.

4.4.5 Unsteady Effects

If we consider that the partial derivative of a function in respect to time is not zero, then the amount of the parameter B may change with time. Then, we take the especial case, that the inflow of

parameter B is equal to the outflow, this will make the integral in eq. 4.19 of the control surface equal zero.

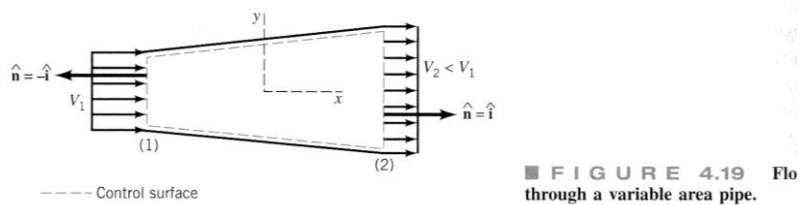
Turns into:



For Example, take figure 4.18, and assuming the flow is onedimensional with $\mathbf{V}=V_0\hat{\mathbf{i}}$, where V_0 is a function of time and that density is constant, we have all particles moving at the same velocity at any instant. Now we take the parameter $B=$ system momentum $=$ (mass)* $\mathbf{V} =$ (mass)* $V_0\hat{\mathbf{i}}$, and thus, $b=$ fluid velocity $=B/(\text{mass}) = \mathbf{V} = V_0\hat{\mathbf{i}}$. The efflux sign is opposite to that of the influx since $\mathbf{V}^*\hat{\mathbf{n}} > 0$ for the outflow and $\mathbf{V}^*\hat{\mathbf{n}} < 0$ for the inflow. From the characteristics that at the sides of the control volume we have the expression $\mathbf{V}^*\hat{\mathbf{n}} = 0$, at section (1) the expression $\mathbf{V}^*\hat{\mathbf{n}} = -V_0$, and at section (2) the expression $\mathbf{V}^*\hat{\mathbf{n}} = V_0$, and that $A_1 = A_2$, we get

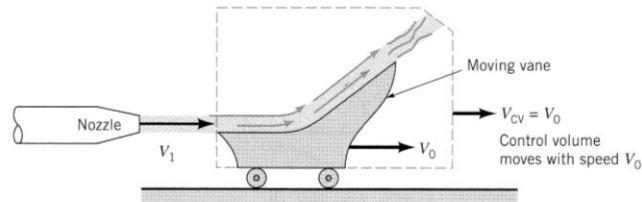
$$\begin{aligned} \int_{cs} \rho b \mathbf{V} \cdot \hat{\mathbf{n}} dA &= \int_{cs} \rho(V_0\hat{\mathbf{i}})(\mathbf{V} \cdot \hat{\mathbf{n}}) dA \\ &= \int_{(1)} \rho(V_0\hat{\mathbf{i}})(-V_0)dA + \int_{(2)} \rho(V_0\hat{\mathbf{i}})(V_0)dA \\ &= -\rho V_0^2 A_1 \hat{\mathbf{i}} + \rho V_0^2 A_2 \hat{\mathbf{i}} = 0 \end{aligned}$$

Eq. 4.21 is satisfied; if V_0 is constant in time, then each term in the Reynolds transport theorem is zero. The following figure (4.19) is an example in which V_1 is no equal to V_2 , the efflux is not equal to the influx, thus the convective term in eq. 4.20 is not zero.



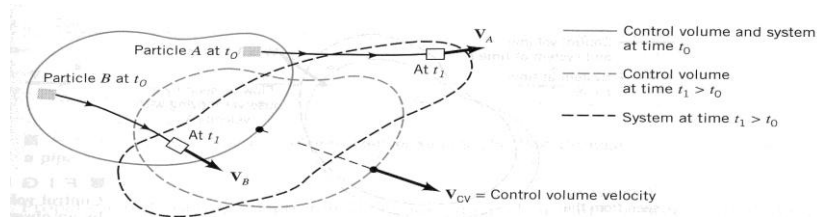
4.4.6 Moving Control Volumes

The control volume is usually static, but sometimes we can allow it to move or deform, so a problem can easily be solve by something more complex. For example, a moving nondeforming control volume as shown in Fig. 4.20, in which a stream of water with velocity V_1 strikes a vane which moves at constant velocity V_0 . We may want to determine the force that the water puts on the vane. We will develop the Reynolds transport theorem for such control volumes.



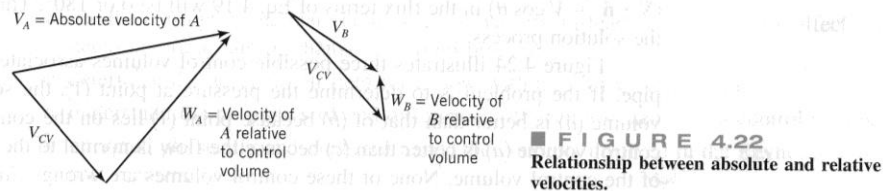
■ FIGURE 4.20 Example of a moving control volume.

Now we shall consider a case on which the control volume moves with a constant velocity, V_{cv} . This control volume does not change properties with time. There is a flow of fluid due to the fact that the velocity of the control volume and that of the fluid are not the same. It is just like the static case with the difference on the relative velocity, W (as seen by an observer on the control volume), that carries the moving control surface, on the other case is the absolute velocity, V (as seen by a static observer), that carries the fluid across the fixed control surface. Thus, their difference is $V_{cv} = V - W$. As shown on Fig. 4.21.



■ FIGURE 4.21 Typical moving control volume and system.

Note: The velocity is a vector, so we must use vector addition to obtain the relative velocity, if we know the other two. Generally, the absolute velocity, V , and the control volume velocity, V_{cv} , will not be in the same direction, so that V and W have different directions.



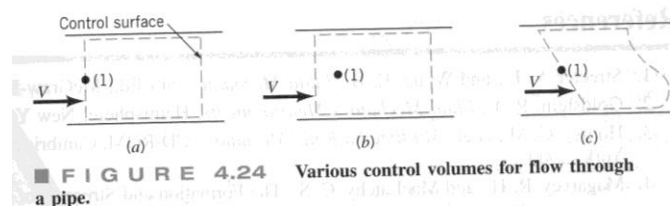
■ FIGURE 4.22 Relationship between absolute and relative velocities.

Now, if we take eq. 4.19 (or Reynolds transport theorem for a fixed control volume), we can derive it for a moving control volume just by replacing V by W , or the absolute for the relative velocity.

$$\frac{DB_{sys}}{Dt} = \frac{\partial}{\partial t} \int_{cv} \rho b dV + \int_{cs} \rho b \mathbf{W} \cdot \hat{\mathbf{n}} dA \quad (4.23)$$

4.4.7 Selection of a Control Volume

There are no wrong Control Volumes; there is just one that makes easier the description of the problem or its solution. We may choose one that is of finite size or infinitesimal in size, although usually fixed and nondeforming, we may consider one that moves at a constant velocity. Thus, it is a good idea to ensure that if we want a parameter at a given point, such point, is at the control surface, not “buried” within the control volume. The unknown will then appear in the convective term (the surface integral) of the Reynolds transport theorem. For simplicity, the control surface should be normal to the fluid velocity so that the angle θ ($V \cdot \hat{n} = V \cos \theta$) on eq. 4.19, is either 0 or 180° . Now, Fig. 4.24 shows three cases of possible control volumes associated with flow through a pipe. Being (a), the best of the three, because the point of the unknown lies on the control surface, and also because the flow is normal to both the inlet and exit portions of the control volume.



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