Group 5

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Chapter 6: Interphase Transport in Isothermal Systems

6.1 DEFINITION OF FRICTION FACTORS

There are two possible systems when considering steady flows with constant density:

a. Fluid flows in straight conduit of uniform cross section.

b. Fluid flows around submerged object that has an axis of symmetry parallel to the direction of the approaching fluid.

$\mathbf{F}_{f\to s} = \mathbf{F}_s + \mathbf{F}_k$ → Additional force associated with fluid motion

- $\mathbf{F}_{f\to s}$: Force exerted by the fluid on the solid surfaces
- $\mathbf{F}_s$: Force that would be exerted by the fluid even if it were stationary

For case (a): $\mathbf{F}_k$ same direction as <**V**> in the conduit

For case (b): $\mathbf{F}_k$ same direction as the approach velocity $\mathbf{V}_\infty$

$\mathbf{F}_k = \mathbf{A\,K\,f}$ → f: Friction factor

- A: Area
- K: Kinetic energy per unit volume

(a) Flow in conduits

$\mathbf{F}_k = \mathbf{A\,K\,f}$

Fanning friction factor

$$f = \frac{1}{4}\left(\frac{D}{L}\right)\left(\frac{\mathcal{P}_0 - \mathcal{P}_L}{\frac{1}{2}\rho\langle v\rangle^2}\right)$$

Wetted surface $\frac{1}{2}\rho \langle V\rangle^2$

(b) Flow around submerged objects

$\mathbf{F_k = A\,K\,f}$

Solid area in a plane perpendicular to V^{∞} ← → $\frac{1}{2}\rho\, \mathbf{V}^{\infty}$

$$f = \frac{4}{3}\frac{gD}{v_\infty^2}\left(\frac{\rho_{sph} - \rho}{\rho}\right)$$

6.2 FRICTION FACTORS FOR FLOW IN TUBES

In this section we have different friction factors for different situations.

For steady-state:

$$f = \frac{4}{3}\frac{gD}{v_\infty^2}\left(\frac{\rho_{sph} - \rho}{\rho}\right)$$

For laminar flow:

$$f = \frac{16}{Re}\begin{Bmatrix} Re < 2100 & \text{stable} \\ Re > 2100 & \text{usually unstable} \end{Bmatrix} \qquad (6.2\text{-}11)$$

For turbulent flows (by experimental data):

Blasius Formula $$f = \frac{0.0791}{Re^{1/4}} \qquad 2.1 \times 10^3 < Re < 10^5 \qquad (6.2\text{-}12)$$

Prandtl Formula $$\frac{1}{\sqrt{f}} = 4.0\ \log_{10} Re\sqrt{f} - 0.4 \qquad 2.3 \times 10^3 < Re < 4 \times 10^6 \qquad (6.2\text{-}13)$$

Barenblatt Formula $$f = \frac{2}{\Psi^{2/(\alpha+1)}} \qquad \text{where} \qquad \Psi = \frac{e^{3/2}(\sqrt{3} + 5\alpha)}{2^{\alpha}\alpha(\alpha + 1)(\alpha + 2)} \qquad (6.2\text{-}14)$$

For turbulent flow in noncircular tubes:

$$R_h = S/Z \qquad (6.2\text{-}16)$$

Where R_h is the "mean hydraulic radius", S is the cross section of the conduit and Z is the wetted perimeter.

Fig 6.2-2 is important for this analysis.

See examples 6.2-1 and 6.2-2.

Chapter 7: Macroscopic Balances for Isothermal Flow Systems

Chapter Introduction (BSL)

Equations of change for isothermal systems were obtained by using conservation laws over a "microscopic system", however, in the example shown below our system is a macroscopic one.

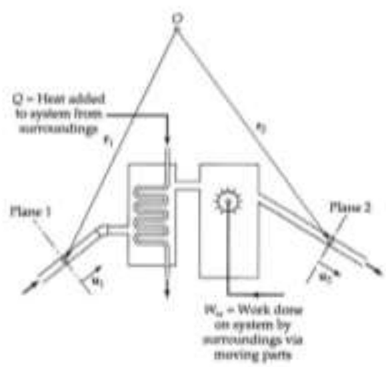

Fig. 7.0-1. Macroscopic flow system with fluid entering at plane 1 and leaving at plane 2. It may be necessary to add heat at a rate Q to maintain the system temperature constant. The rate of doing work on the system by the surroundings by means of moving surfaces is W_m. The symbols $\mathbf{u}_1$ and $\mathbf{u}_2$ denote unit vectors in the direction of flow at planes 1 and 2. The quantities $\mathbf{r}_1$ and $\mathbf{r}_2$ are position vectors giving the location of the centers of the inlet and outlet planes with respect to some designated origin of coordinates.

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For this system macroscopic balance are used. The macroscopic balances contain terms that account for the interactions of the fluid with the solid surfaces. The fluid can exert forces and torques on the system's surface, and the surroundings can do work W_m on the fluid by means of moving surfaces. These are the macroscopic balances:

$$\int_{V(t)} (\text{eq. of continuity})\, dV = \text{macroscopic mass balance}$$

$$\int_{V(t)} (\text{eq. of motion})\, dV = \text{macroscopic momentum balance}$$

$$\int_{V(t)} (\text{eq. of angular momentum})\, dV = \text{macroscopic angular momentum balance}$$

$$\int_{V(t)} (\text{eq. of mechanical energy})\, dV = \text{macroscopic mechanical energy balance}$$

For macroscopic balances sometimes when seeing the behavior of the problem, one can assumed certain values and therefore, cancelling some of the terms. This requires;

- Intuition, based on experience with similar systems
- Some experimental data on the system
- Flow visualization studies
- Order of magnitude estimates

7.1 THE MACROSCOPIC MASS BALANCE

In the system shown in fig 7.0-1 the fluid enters the system at plane 1 with cross section S_1 and leaves at plane 2 with cross sectionS_2, respectively the average velocities are $\langle v_1\rangle$ and $\langle v_2\rangle$. Two assumptions not so restrictive are introduced:

1. at the planes 1 and 2 the time smoothed velocity is perpendicular to the relevant cross section
2. at planes 1 and 2 the density and other physical properties are uniform over the cross section.

For this system the law of conservation of mass is:

$$\frac{d}{dt} m_{\text{tot}} = \rho_1\langle v_1\rangle S_1 - \rho_2\langle v_2\rangle S_2 \qquad (7.1\text{-}1)$$

rate of increase of mass	rate of mass in at plane 1	rate of mass out at plane 2

At this equation $m_{tot} = \int \rho dV$ is the total mass of fluid contained in the system between planes 1 and 2. The mass rate of flow is $w = \rho\langle v\rangle S$ and the notation$\Delta w = w_2 - w_1$. Then the *unsteady-state macroscopic mass balance* becomes

$$\boxed{\frac{d}{dt} m_{\text{tot}} = -\Delta w} \qquad (7.1\text{-}2)$$

For *steady-state* (not change with time)*:*

$$\Delta w = 0 \qquad (7.1-3)$$

See Example 7.1-1 Draining of a Spherical Tank

7.2 THE MACROSCOPIC MOMENTUM BALANCE

For the fig 7.0-1 we stated two important assumptions, now we also introduce two additional assumptions:

3. The forces associated with the stress tensor $\boldsymbol{\tau}$ are neglected at planes 1 and 2, since they are generally small compared to the pressure forces at the entry and exit planes.
4. The pressure does not vary over the cross section at the entry and exit planes.

$\boldsymbol{u_1}$ and $\boldsymbol{u_2}$ represents in the following equation (law of conservation of momentum) the direction of flow at planes 1 and 2.

$$\frac{d}{dt}\mathbf{P}_{\text{tot}} = \rho_1\langle v_1^2\rangle S_1\mathbf{u}_1 - \rho_2\langle v_2^2\rangle S_2\mathbf{u}_2 + p_1S_1\mathbf{u}_1 - p_2S_2\mathbf{u}_2 + \mathbf{F}_{s\to f} + m_{\text{tot}}\mathbf{g} \qquad (7.2\text{-}1)$$

rate of increase of momentum	rate of momentum in at plane 1	rate of momentum out at plane 2	pressure force on fluid at plane 1	pressure force on fluid at plane 2	force of solid surface on fluid	force of gravity on fluid

Where $P_{tot} = \int \rho v dV$ is the total momentum in the system. The subscripts "$s \to f$" serves as a reminder of the direction of the force.

By introducing for the mass flow rate and Δ symbol we obtain the *unsteady-state macroscopic momentum balance*

$$\frac{d}{dt}\mathbf{P}_{tot} = -\Delta\left(\frac{\langle v^2\rangle}{\langle v\rangle}w + pS\right)\mathbf{u} + \mathbf{F}_{s\to f} + m_{tot}\mathbf{g} \qquad (7.2\text{-}2)$$

As the same way, if the total amount of momentum in the system does not change with time, we obtained the *steady-state macroscopic momentum balance*,

$$\mathbf{F}_{f\to s} = -\Delta\left(\frac{\langle v^2\rangle}{\langle v\rangle}w + pS\right)\mathbf{u} + m_{tot}\mathbf{g} \qquad (7.2\text{-}3)$$

It is important to know several changes for turbulent flow;

- $\langle v\rangle \longrightarrow \langle \bar{v}\rangle$ and $\langle v^2\rangle \to \langle \bar{v}^2\rangle$
- $\langle \bar{v}^2\rangle / \langle \bar{v}\rangle \to \langle \bar{v}\rangle$
- Drop angular brackets and over bars.

See Example 7.1-1 Force Exerted by a Jet (Part a)

7.1-1 Assumptions

- Jet has a constant radius, R_1 , between the tube exit and the disk.
- Jet spreads slightly
- Pressure is atmospheric at both planes
- z Component of the fluid velocity at plane 2 is zero.

The momentum balance (example 7.1-1) then becomes,

$$mg = v_1(\rho v_1 \pi R_1^2) - (\pi R_1^2 h)\rho g \qquad (7.2\text{-}4)$$

For h,

$$h = \frac{v_1^2}{g} - \frac{m}{\rho \pi R_1^2} = \frac{(6)^2}{(9.807)} - \frac{5.5}{\pi(0.025)^2} = 0.87 \text{ m} \qquad (7.2\text{-}5)$$

7.3 THE MACROSCOPIC ANGULAR MOMENTUM BALANCE

The difference between linear and angular momentum balance are the following:

- Replace “momentum” by “angular momentum”.
- “force” by “torque”

In Fig. 7.0-1 the origin now can be designated as “O” and the locations of the midpoints of planes 1 and 2 with respect to this origin are given by the position vector $\boldsymbol{r_1}$ and $\boldsymbol{r_2}$. Using the assumptions stated previously; the *unsteady-state macroscopic angular momentum balance* can be written as,

$$\frac{d}{dt}\mathbf{L}_{tot} = \rho_1\langle v_1^2\rangle S_1[\mathbf{r}_1 \times \mathbf{u}_1] - \rho_2\langle v_2^2\rangle S_2[\mathbf{r}_2 \times \mathbf{u}_2]$$

rate of increase of angular momentum; rate of angular momentum in at plane 1; rate of angular momentum out at plane 2

$$+ p_1S_1[\mathbf{r}_1 \times \mathbf{u}_1] - p_2S_2[\mathbf{r}_2 \times \mathbf{u}_2] + \mathbf{T}_{s \to f} + \mathbf{T}_{ext} \qquad (7.3\text{-}1)$$

torque due to pressure on fluid at plane 1; torque due to pressure on fluid at plane 2; torque of solid surface on fluid; external torque on fluid

On this equation $L_{tot} = \int \rho[r \times v]dV$ is the total angular momentum and $T_{ext} = \int[r \times \rho g]dV$ is the torque on the fluid. The equation can be also written,

$$\boxed{\frac{d}{dt}\mathbf{L}_{tot} = -\Delta\left(\frac{\langle v^2\rangle}{\langle v\rangle}w + pS\right)[\mathbf{r} \times \mathbf{u}] + \mathbf{T}_{s \to f} + \mathbf{T}_{ext}} \qquad (7.3\text{-}2)$$

Steady-state macroscopic angular momentum balance

$$\mathbf{T}_{f \to s} = -\Delta\left(\frac{\langle v^2\rangle}{\langle v\rangle}w + pS\right)[\mathbf{r} \times \mathbf{u}] + \mathbf{T}_{ext} \qquad (7.3\text{-}3)$$

See example 7.3-1 for better understanding.

7.4 THE MACROSCOPIC MECHANICAL ENERGY BALANCE

First, we must integrate the equation of change of mechanical energy over the volume of the flow system. Using the four assumptions we obtain the *unsteady-state macroscopic mechanical energy balance* (*engineering Bernoulli equation*).

$$\frac{d}{dt}(K_{tot} + \Phi_{tot}) = \left(\tfrac{1}{2}\rho_1\langle v_1^3\rangle + \rho_1\hat{\Phi}_1\langle v_1\rangle\right)S_1 - \left(\tfrac{1}{2}\rho_2\langle v_2^3\rangle + \rho_2\hat{\Phi}_2\langle v_2\rangle\right)S_2$$

rate of increase of kinetic and potential energy in system; rate at which kinetic and potential energy enter system at plane 1; rate at which kinetic and potential energy leave system at plane 2

$$+ (p_1\langle v_1\rangle S_1 - p_2\langle v_2\rangle S_2) + W_m + \int_{V(t)} p(\nabla \cdot \mathbf{v})\,dV + \int_{V(t)} (\boldsymbol{\tau}:\nabla\mathbf{v})\,dV \qquad (7.4\text{-}1)$$

net rate at which the surroundings do work on the fluid at planes 1 and 2 by the pressure; rate of doing work on fluid by moving surfaces; rate at which mechanical energy increases or decreases because of expansion or compression of fluid; rate at which mechanical energy decreases because of viscous dissipation

On this equation $K_{tot} = \frac{1}{2}\int \rho v^2 dV$ and $\Phi_{tot} = \int \rho\hat{\Phi}dV$ are the total kinetic and potential energies within the system. At the system entrance the force p_1S_1 multiplied by the velocity $\langle v_1\rangle$ gives the rate at which the surroundings do work on the fluid. Now our equation can be written as;

$$\boxed{\frac{d}{dt}(K_{tot} + \Phi_{tot}) = -\Delta\left(\frac{1}{2}\frac{\langle v^3\rangle}{\langle v\rangle} + \hat{\Phi} + \frac{p}{\rho}\right)w + W_m - E_c - E_v} \qquad (7.4\text{-}2)$$

in which the terms E_c and E_v are defined as follows:

$$E_c = -\int_{V(t)} p(\nabla \cdot \mathbf{v})\,dV \quad \text{and} \quad E_v = -\int_{V(t)} (\boldsymbol{\tau}:\nabla\mathbf{v})\,dV \qquad (7.4\text{-}3, 4)$$

For this equation E_c (Compression term) is positive in a compression and negative in expansion; when a fluid is incompressible this term is zero. E_v (Viscous dissipation, also known as friction loss term) is always positive for Newtonian liquids.

For *steady-state macroscopic mechanical energy balance* is,

$$\Delta\left(\frac{1}{2}\frac{\langle v^3\rangle}{\langle v\rangle} + gh + \frac{p}{\rho}\right)w = W_m - E_c - E_v \qquad (7.4\text{-}5)$$

We can get the next equation by drawing a representative streamline through the system. Combining $\Delta(p/\rho)$ and E_c we can the following approximate relation,

$$\Delta\left(\frac{p}{\rho}\right)w + E_c \approx w\int_1^2 \frac{1}{\rho}\,dp \qquad (7.4\text{-}6)$$

Then, after dividing Eq. 7.4-5 by $w_1 = w_2 = w$, we get

$$\Delta\left(\frac{1}{2}\frac{\langle v^3\rangle}{\langle v\rangle}\right) + g\Delta h + \int_1^2 \frac{1}{\rho}\,dp = \hat{W}_m - \hat{E}_v \qquad (7.4\text{-}7)$$

Where $\widehat{W}_m = W_m/w$ and $\hat{E}_v = E_v/w$. For isothermal systems, the integral term can be calculated as long the expression for density as function of pressure is available.For turbulent flow we adjust the steps stated in section 7.2.

See example 7.3-1(Part b) for better understanding.

7.5 ESTIMATION OF VISCOUS LOSS

E$_v$, which appears in the macroscopic mechanical energy balance. The general expression for E$_v$:$\boldsymbol{E_v = -\int(\tau : \bar{V} v dV)}$

and using equation 3.3-3 for incompressible Newtonian we can rewrite E$_v$ (7-5.1):

$$E_v = \int \mu \Phi_v dV$$

which shows that it is the integral of the local rate viscous dissipation over the volume of the entire flow system.

The quantities Φ_v is a sum of squares velocity gradients; hence it has dimensions of $(v_0/l_0)^2$, where v_0 and l_0 are the characteristic velocity and length, respectively. We can therefore write (7.5-2):

$$E_v - \left(\mu {v_0}^3 {l_0}^2\right)(\mu/v_0 l_0 \rho) \int \breve{\Phi}_v d\breve{V}$$

where $\breve{\Phi}_v = (v_0/l_0)^2 \Phi_v$ and $d\breve{V} = l_0{}^{-3} dV$ are dimensionless quantities. Hence, if the only significant dimensionless group is a Reynolds number, $Re = v_0 l_0 \rho/\mu$, then Eq. 7.5-2 most have the general form (7.5-3):

$$E_v - (\rho v_0{}^3 l_0{}^2) \times \begin{pmatrix} a\ dimensionless\ function\ of\ Re \\ and\ various\ geometrical\ ratios \end{pmatrix}$$

In steady state flow we prefer to work with the quantity $\hat{E}_v = E_v/w$, in which $w = \rho\langle v\rangle S$ is the mass rate of flow passing through any cross section of the flow system. If we select the reference velocity v_0 to be $\langle v\rangle$ and the reference length l_0 to be $\sqrt{S}$, then (7.5-4)

$$\hat{E}_v = 1/2\langle v\rangle^2 e_v$$

In which e_v, the friction loss factor, is a function of a Reynolds numbers and relevant dimensionless geometrical ratios

For a straight conduit the friction loss factor is closely related to the friction factor. We consider only the steady flow of a fluid of constant density in a straight conduit of arbitrary, but constant, cross section S and length L. If the fluid is flowing in the z direction under the influence of a pressure gradients, then Eqs 7.2-2 and 7.4-7 become:

$$(z\ momentum) \qquad F_{f\to S} = (p_1 - p_2)S + (\rho SL)g_z \qquad (7.5-5)$$

$$(mechanical\ energy) \qquad \hat{E}_v = \frac{1}{\rho(p_1 - p_2)} + L \qquad (7.5-6)$$

Multiplication of the second of these by ρS and subtracting gives (7.5-7)

$$\hat{E}_v = \frac{F_{f\to S}}{\rho S}$$

If in addition the flow is turbulent then the expression for $F_{f\to S}$ in terms of the mean hydraulic radius R_h may be used so that (7.5-8)

$$\hat{E}_v = 1/2\langle v\rangle^2 \frac{L}{R_h} f$$

in which *f* is the friction factor discussed on chapter 6. Since this equation is in the form of Eq 7.5-4, we get a simple relation between the friction loss factor and the friction factor (7.5-9)

$$e_v = \frac{L}{R_h} f$$

for turbulent flow the sections of straight pipe with uniform cross section. For a similar treatment for conduits of variable cross section, see Problem 7B.2.

Most flow system contains various "obstacle," such as fittings, sudden changes in diameter, valves, or flow measuring devices. These also contribute to the friction loss $\hat{E}_v$. Such additional resistances may be written in the form of Eq. 7.5-4 with e_v determine by one of two methods: (a) simultaneous solution of the macroscopic balances, or (b) experimental measurement. Some rough values of e_v are tabulate in table 7.5-1 for the convention that $\langle v \rangle$ is the average velocity downstream from the disturbance.

Now we are in the position to rewrite Eq. 7.4-7 in the approximate form frequently used for turbulent flow calculations in a system composed of various kinds of piping and additional resistances (7.5-10):

$$\frac{1}{2}({v_2}^2 - {v_1}^2) + g(z_2 - z_1) + \int_{p_1}^{p_2} \frac{1}{\rho} dp = \widehat{W}_m - \sum_i (\frac{1}{2} v^2 \frac{L}{R_h} f) - \sum_i (\frac{1}{2} v^2 e_v)_i$$

Here R_h is the mean hydraulic radius defined in Eq. 6.2 -16, *f* is the friction factor defined in 6.1-4, and e_v is the friction loss factor given in table 7.5-1. Note that the $v_1$and v_2 in the first term reference to the velocity at planes 1 and 2, the *v* in the first sum is the average velocity downstream from the *i*th fitting, valve, or other obstacle.

7.6 USE OF MACROSCOPIC BALANCES FOR STEADY STATE PROBLEMS

This section will be dedicated to explaining different problems and how to get quantities such as pressure drops, friction losses, and force on the pipe.

7.6-1 An incompressible fluid flows from a small circular tube to a large tube with turbulent flow, as shown in figure 7.6-1. The cross sectional areas of the tubes are S_1 and S_2. Obtain the expretion for the pressure change from plane 1 and 2 and for the friction loss associated with the sudden enlargement of cross section. Let $\beta = S_1/S_2$, which is less than unity.

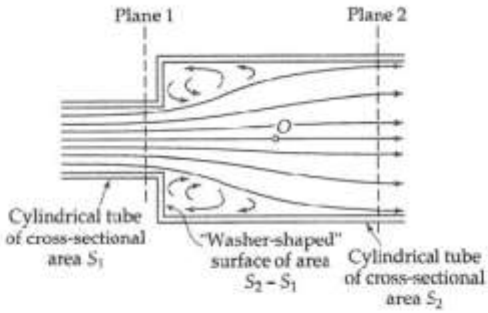

Fig. 7.6-1. Flow through a sudden enlargement.

(a) Mass balance For steady state the mass balance gives

$$w_1 = w_2 \quad \text{or} \quad \rho_1 v_1 S_1 = \rho_2 v_2 S_2 \qquad (7.6\text{-}1)$$

For a fluid of constant density this gives:

$$\frac{v_1}{v_2} = \frac{1}{\beta} \qquad (7.6\text{-}2)$$

(b) Momentum balance The downstream component of the momentum balance is:

$$\mathbf{F}_{f\to s} = (v_1 w_1 - v_2 w_2) + (p_1 S_1 - p_2 S_2) \qquad (7.6\text{-}3)$$

The force $\mathbf{F}_{f\to s}$ has two parts: one is the viscous forces excerted my the fluid on the sides of the pipes in the direction of the flow, and the other is the pressure exerted on the washer-shaped surface just to the right of plane 1 and perpendicular to the flow axis.

The viscous one we neglect by intuition, and the other one we take to be $p_1(S_2 - S_1)$ by assuming the pressure in this area is the same as that in plane 1. Then we get by using eq (7.6-1):

$$-p_1(S_2 - S_1) = \rho v_2 S_2(v_1 - v_2) + (p_1 S_1 - p_2 S_2) \qquad (7.6\text{-}4)$$

Solving for the pressure difference gives:

$$p_2 - p_1 = \rho v_2(v_1 - v_2) \qquad (7.6\text{-}5)$$

Or, in terms of the downstream velocity:

$$p_2 - p_1 = \rho v_2^2\left(\frac{1}{\beta} - 1\right) \qquad (7.6\text{-}6)$$

Note thet it correctly predicts a rise in pressure

(c) Angular momentum balance This balance is not needed since no torque will be exerted on the system

(d) Mechanical energy balance Since there is no compressive loss, no work done by moving parts, and no change in elevation:

$$\hat{E}_v = \tfrac{1}{2}(v_1^2 - v_2^2) + \frac{1}{\rho}(p_1 - p_2) \tag{7.6-7}$$

Substituting (7.6-6) for the change in pressure and after some rearangement gives:

$$\hat{E}_v = \tfrac{1}{2}v_2^2\left(\frac{1}{\beta} - 1\right)^2 \tag{7.6-8}$$

Notice however that to get to this equation of the friction loss, many assumpirions were made. If greater accuracy is needed, a correction factor based on experimental data should be introduced.

7.6-2 A diagram of a liquid-liquid ejector is shown in Figure 7.6-2. It is desired to analyze the mixing of the two streams, both of the same fluid, using macroscopic balances. At plane one the two fluid streams merge. Stream 1a has a velocity of v_0 and a cross sectional area of $\frac{1}{3}S_1$, and stream 1b has a velocity of $\frac{1}{2}v_0$ and a cross sectional area of $\frac{2}{3}S_1$. Plane 2 is chosen far enough downstream so that the streams are mixed and the velocity is uniform at v_2. The flow is turbulent and the velocity profiles at planes 1 and 2 are asummed to be flat. In the following analysis $\mathbf{F}_{f\to s}$ is neglected because it is felt to be less important than other terms in the momentum balance.

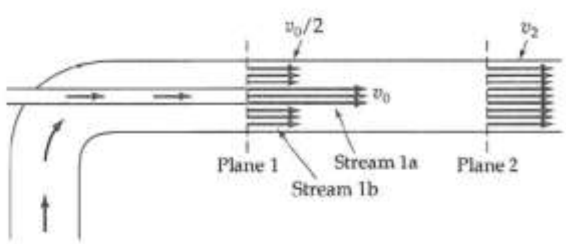

Fig. 7.6-2. Flow in a liquid–liquid ejector pump.

(a) Mass balance At steady state:

$$w_{1a} + w_{1b} = w_2 \tag{7.6-9}$$

Or

$$\rho v_0(\tfrac{1}{3}S_1) + \rho(\tfrac{1}{2}v_0)(\tfrac{2}{3}S_1) = \rho v_2 S_2 \tag{7.6-10}$$

Since $S_1 = S_2$ then:

$$v_2 = \tfrac{2}{3}v_0 \tag{7.6-11}$$

(b) Momentum balance The component of the momentum balance in the direction of flow is:

$$(v_{1a}w_{1a} + v_{1b}w_{1b} + p_1S_1) - (v_2w_2 + p_2S_2) = 0 \qquad (7.6\text{-}12)$$

Or using the relation at the end of (a)

$$\begin{aligned}(p_2 - p_1)S_2 &= (\tfrac{1}{2}(v_{1a} + v_{1b}) - v_2)w_2 \\ &= (\tfrac{1}{2}(v_0 + \tfrac{1}{2}v_0) - \tfrac{2}{3}v_0)(\rho(\tfrac{2}{3}v_0)S_2)\end{aligned} \qquad (7.6\text{-}13)$$

From which:

$$p_2 - p_1 = \tfrac{1}{18}\rho v_0^2 \qquad (7.6\text{-}14)$$

This correctly predicts a rise in pressure

(c) Angular momentum balance This balance is not needed

(d) Mechanical energy balance:

$$(\tfrac{1}{2}v_{1a}^2w_{1a} + \tfrac{1}{2}v_{1b}^2w_{1b}) - \left(\tfrac{1}{2}v_2^2 + \frac{p_2 - p_1}{\rho}\right)w_2 = E_v \qquad (7.6\text{-}15)$$

Using the relation at the end of (a) we get:

$$(\tfrac{1}{2}v_{1a}^2(\tfrac{1}{2}w_2) + \tfrac{1}{2}(\tfrac{1}{2}v_0)^2(\tfrac{1}{2}w_2)) - (\tfrac{1}{2}(\tfrac{2}{3}v_0)^2 + \tfrac{1}{18}v_0^2)w_2 = E_v \qquad (7.6\text{-}16)$$

Hence:

$$\hat{E}_v = \frac{E_v}{w_2} = \frac{5}{144}v_0^2 \qquad (7.6\text{-}17)$$

which is the energy disipation per unit mass.

7.6-3 Water at 95° C is flowing at a rate of 2.0 ft^3/s through a 60° bend, in which there is a contraction from 4 to 3 in. internal diameter (See figure 7.6-3). Compute the pressure exerted on the bend if the pressure at the downstream end is 1.1 atm. The density and viscosity of the water at the conditions of the system are 0.962 g/cm^3 and 0.299 cp respectively.

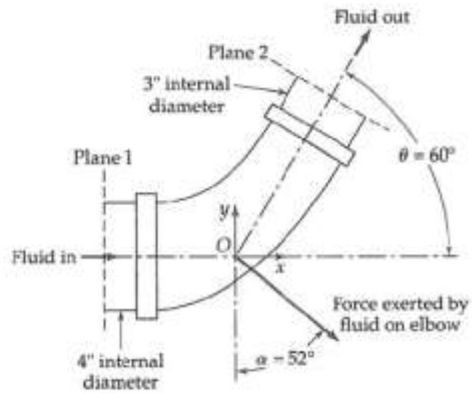

Fig. 7.6-3. Reaction force at a reducing bend in a pipe.

The Reynolds number fot the flow in the 3 in end is:

$$\text{Re} = \frac{D\langle v\rangle\rho}{\mu} = \frac{4w}{\pi D\mu}$$
$$= \frac{4(2.0 \times (12 \times 2.54)^3)(0.962)}{\pi(3 \times 2.54)(0.00299)} = 3 \times 10^6 \qquad (7.6\text{-}18)$$

At this Reynolds number the flow is highly turbulent and the assumption of the flat velocity profile is reasonable.

(a) Mass balance for steady state and constant density:

$$\frac{v_1}{v_2} = \frac{S_2}{S_1} \equiv \beta \qquad (7.6\text{-}19)$$

(b) Mechanical energy balance:

$$\tfrac{1}{2}(v_2^2 - v_1^2) + g(h_2 - h_1) + \frac{1}{\rho}(p_2 - p_1) + \hat{E}_v = 0 \qquad (7.6\text{-}20)$$

According to Table 7.5-1 and Eq. 7.5-4. We can take the friction loss as approximately $\frac{2}{5}(\frac{1}{2}v_2^2) = \frac{1}{5}v_2^2$. Inserting this into Eq. 7.6-20 and using the mass balance we get:

$$p_1 - p_2 = \rho v_2^2(\tfrac{1}{2} - \tfrac{1}{2}\beta^2 + \tfrac{1}{5}) + \rho g(h_2 - h_1) \qquad (7.6\text{-}21)$$

This is the pressure drop through the bend in terms of the known velocity v_2 and the known geometrical factor β

(c) Momentum balance We now have to consider both the x- and y- components of the momentum balance. The inlet and outlet unit vectors will have x- and y- components given by $u_{1x} = 1$, $u_{1y} = 0$, $u_{2x} = \cos\theta$, and $u_{2y} = \sin\theta$.

The x- component of the momentum balance gives:

$$F_x = (v_1 w_1 + p_1 S_1) - (v_2 w_2 + p_2 S_2)\cos\theta \qquad (7.6\text{-}22)$$

Where F_x is the x- component of $\mathbf{F}_{f\to s}$. Introducing the specific expressions for w_1 and w_2 we get

$$\begin{aligned} F_x &= v_1(\rho v_1 S_1) - v_2(\rho v_2 S_2)\cos\theta + p_1 S_1 - p_2 S_2 \cos\theta \\ &= \rho v_2^2 S_2(\beta - \cos\theta) + (p_1 - p_2)S_1 + p_2(S_1 - S_2\cos\theta) \end{aligned} \qquad (7.6\text{-}23)$$

Substituting into this expression for $p_1 - p_2$ from Eq. 7.6-21 gives

$$\begin{aligned} F_x &= \rho v_2^2 S_2(\beta - \cos\theta) + \rho v_2^2 S_2 \beta^{-1}(\tfrac{7}{10} - \tfrac{1}{2}\beta^2) \\ &\quad + \rho g(h_2 - h_1)S_2\beta^{-1} + p_2 S_2(\beta^{-1} - \cos\theta) \\ &= w^2(\rho S_2)^{-1}(\tfrac{7}{10}\beta^{-1} - \cos\theta + \tfrac{1}{2}\beta) \\ &\quad + \rho g(h_2 - h_1)S_2\beta^{-1} + p_2 S_2(\beta^{-1} - \cos\theta) \end{aligned} \qquad (7.6\text{-}24)$$

The y- component of the momentum balance is:

$$F_y = -(v_2 w_2 + p_2 S_2)\sin\theta - m_{\text{tot}}g \qquad (7.6\text{-}25)$$

or

$$F_y = -w^2(\rho S_2)^{-1}\sin\theta - p_2 S_2 \sin\theta - \pi R^2 L \rho g \qquad (7.6\text{-}26)$$

In which R and L are the radius and length of a roghly equivalent cylinder.

Substituting the values we know we get:

$$\begin{aligned} F_x &= \frac{(120)^2}{2(0.049)(32.2)}\left(\frac{7}{10}\frac{1}{0.562} - \frac{1}{2} + \frac{0.562}{2}\right) + (60)(\tfrac{1}{2})(0.049)\left(\frac{1}{0.562}\right) \\ &\quad + (16.2)(0.049)(144)\left(\frac{1}{0.562} - \frac{1}{2}\right)\text{lb}_f \\ &= (152)(1.24 - 0.50 + 0.28) + 2.6 + (144)(1.78 - 0.50) \\ &= 155 + 2.6 + 146 = 304\ \text{lb}_f = 1352\text{N} \end{aligned} \qquad (7.6\text{-}27)$$

$$\begin{aligned} F_y &= -\frac{(120)^2}{2(0.049)(32.2)}\left(\frac{1}{2}\sqrt{3}\right) - (16.2)(0.049)(144)\left(\frac{1}{2}\sqrt{3}\right) - \pi(\tfrac{1}{8})^2(\tfrac{5}{6})(60)\ \text{lb}_f \\ &= -132 - 99 - 2.5 = 234\ \text{lb}_f = 1041\ \text{N} \end{aligned} \qquad (7.6\text{-}28)$$

And hence the maginitud of the force is:

$$|\mathbf{F}| = \sqrt{F_x^2 + F_y^2} = \sqrt{304^2 + 234^2} = 384\ \text{lb}_f = 1708\ \text{N} \qquad (7.6\text{-}29)$$

The angle that this force makes with the vertical line is given by:

$$\alpha = \arctan(F_x/F_y) = \arctan 1.30 = 52^\circ$$

7.7 USE OF THE MACRSOCOPIC BALANCES FOR UNSTEADY-STATE PROBLEMS

In the preceding section we have illustrated the use of the macroscopic balance for solving unsteady-state problems. In this section we turn our attention to unsteady-state problems. We give two examples to illustrate the use of the time dependent macroscopic balance equations.

See for better understanding Example 7.7-1 and Example 7.7-2

7.8 DERIVATION OF THE MACROSCOPIC MECHANICAL ENERGY BALANCE

In Eq. 7.4-2 the macroscopic mechanical energy balance was presented without proof. In this section we show how the equation is obtained by integrating the equation of change for mechanical energy (Eq. 3.3-2) over the entire volume of the flow system of Fig. 7.0-1.

We begin by doing the formal integration:

$$\int_{V(t)} \frac{\partial}{\partial t}\left(\tfrac{1}{2}\rho v^2 + \rho\hat{\Phi}\right) dV = -\int_{V(t)} \left(\nabla \cdot \left(\tfrac{1}{2}\rho v^2 + \rho\hat{\Phi}\right)\mathbf{v}\right) dV - \int_{V(t)} (\nabla \cdot p\mathbf{v})\, dV - \int_{V(t)} (\nabla \cdot [\boldsymbol{\tau} \cdot \mathbf{v}])\, dV$$
$$+ \int_{V(t)} p(\nabla \cdot \mathbf{v})\, dV + \int_{V(t)} (\boldsymbol{\tau} : \nabla \mathbf{v})\, dV \qquad (7.8\text{-}1)$$

Next we apply the 3-dimensional Leibniz formula (Eq. A.5-5) to the left side and the Gauss divergence theorem (Eq. A.5-2) to terms 1,2 and 3 on the right sides.

$$\frac{d}{dt}\int_{V(t)} \left(\tfrac{1}{2}\rho v^2 + \rho\hat{\Phi}\right) dV = -\int_{S(t)} \left(\mathbf{n} \cdot \left(\tfrac{1}{2}\rho v^2 + \rho\hat{\Phi}\right)(\mathbf{v} - \mathbf{v}_S)\right) dS - \int_{S(t)} (\mathbf{n} \cdot p\mathbf{v})\, dS$$
$$- \int_{S(t)} (\mathbf{n} \cdot [\boldsymbol{\tau} \cdot \mathbf{v}])\, dS + \int_{V(t)} p(\nabla \cdot \mathbf{v})\, dV + \int_{V(t)} (\boldsymbol{\tau} : \nabla \mathbf{v})\, dV \qquad (7.8\text{-}2)$$

The term containing v_s, the velocity of the surface of the system, arises from the application of the Leibniz formula. The surface *S(t)* consists of four parts.

- The fixed surface S_f (on which both v and v_s are zero)
- The moving surfaces S_m (on which $v = v_s$ with both nonzero)
- The cross section of the entry port S_1 (where v_s=0)
- The cross section of the exit port S_2 (where v_s= 0)

Presently each of the surface integrals will be split into four parts corresponding to these four surfaces.

We now interpret the terms in Eq. 7.8-2 and, in the process, introduce several assumptions; these assumptions have already been mentioned in sec 7.1 to 7.4, but now the reason for them will be made clear.

the term on the left side can be interpreted as the time rate of change of the total kinetic and potential energy $(K_{tot} + \Phi_{tot})$within the "control volume," whose shape and volume are changing with time.

We next examine one by one the five terms on the right side:

Term1 (including the minus sing) contributes only at the entry and exit ports and gives the rates of influx and efflux of kinetic and potential energy:

$$\text{Term 1} - (\tfrac{1}{2}\rho_1\langle v_1^3\rangle S_1 + \rho_1\hat{\Phi}_1\langle v_1\rangle S_1) - (\tfrac{1}{2}\rho_2\langle v_2^3\rangle S_2 + \rho_2\hat{\Phi}_2\langle v_2\rangle S_2) \qquad (7.8\text{-}3)$$

The angular brackets is average over the cross section. Assume that the fluid density and potential energy per unit mass are constant over the corss section, and that the fluis is flowing parallel to the tube walls at the entry and exit ports. The first term in Eq. 7.8-3 is positive, since at plane 1,$(-n \cdot v) = \big(u_1 \cdot (u_1 v_1)\big) - v_1$, and the second term is negative, since at plane 2, $(n \cdot v) = \big(u_2 \cdot (u_2 v_2)\big) - v_2$.

Term 2 (including the minus sign) gives no contribution on S_f since v is zero there. On each surface element *dS* of S_m there is a force –npdS acting on a surface moving with a velocity v, and the dot product of these quantities gives the rate at which the surroundings do work on the fluid through the moving surface element *dS*. We use the symbol $W_m^{(p)}$ to indicate the sum of all the surface terms. Furthermore, the integrals over the stationary surface S_1 and S_2 give the work required to push the fluid into the system at plane 1 minus the work required to push the fluid out of the system at plane 2. Therefore term 2 finally gives

$$\text{Term 2} = p_1\langle v_1\rangle S_1 - p_2\langle v_2\rangle S_2 + W_m^{(p)} \qquad (7.8\text{-}4)$$

Here we have assumed that pressure does no contribution on S_1 since v is zero there. The integral over S_m can be interpreted as the rate at which the surrounding do work on the fluid by means of the viscous forces, and this integral is designated as $W_m^{(p)}$. At the entry and exit ports it is conventional to neglect the work terms associates with the viscous forces, since they are generally quite small compared with the pressure contributions. Therefore we get

$$\text{Term 3} = W_m^{(\tau)} \qquad (7.8\text{-}5)$$

We now introduce the symbol $W_m = W_m^{(p)} + W_m^{(\tau)}$ to represent the total rate at which the surroundings do work on the fluid within the system through the agency of the moving surfaces.

Terms 4 and 5 cannot be further simplified, and hence we define

$$\text{Term 4} = +\int_{V(t)} p(\nabla \cdot \mathbf{v})\, dV = -E_c \qquad (7.8\text{-}6)$$

$$\text{Term 5} = +\int_{V(t)} (\boldsymbol{\tau}:\nabla \mathbf{v})\, dV = -E_v \qquad (7.8\text{-}7)$$

For Newtonian fluids the viscous loss E_v is the rate at which mechanical energy is irreversibly degraded into thermal energy because of the viscosity of the fluid and is always a positive quantity (see Eq. 3.3-3). We have already discussed methods for estimating E_v in sec 7.5 the compression term E_c is the rate at which mechanical energy is reversibly changed into thermal energy because of the compressibility of the fluid; it may be either positive or negative. If the fluid being regarded as incompressible, then E_c is zero.

When all the contributions are insert into Eq. 7.8-2 we finally obtain macroscopic mechanical energy balance:

$$\frac{d}{dt}(K_{tot} + \Phi_{tot}) = (\tfrac{1}{2}\rho_1\langle v_1^3\rangle S_1 + \rho_1\hat{\Phi}_1\langle v_1\rangle S_1 + p_1\langle v_1\rangle S_1) - (\tfrac{1}{2}\rho_2\langle v_2^3\rangle S_2 + \rho_2\hat{\Phi}_2\langle v_2\rangle S_2 + p_2\langle v_2\rangle S_2) + W_m - E_c - E_v \qquad (7.8\text{-}8)$$

If, now we introduce the symbols $w_1 = \rho_1\langle v_1\rangle S_1$ and $w_2 = \rho_2\langle v_2\rangle S_2$ for the mass rates of flow in and out, then Eq. 7.8-8 can be rewritten in the form of Eq. 7.4-2. Several assumptions have been made in this development, but normally they are not serious. If the situation warrants, one can go back and include the neglected effects.

It should be noted that the above derivation of the mechanical energy balance does not require that the system be isothermal. Therefore, the results in Eqs. 7.4-2 and 7.8-8 are valid for non isothermal systems.

To get the mechanical energy balance in the form of Eq. 7.4-7 we have to develop an approximate expression for E_c. We imagine that there is a representative streamline running through the system, and we introduce a coordinate s along the streamline. We assume that pressure, density and velocity do not vary over the cross section. We further imagine that at each position along the streamline, there is a cross section S(s) perpendicular to the s-coordinate, so that we can write *dV=S(s)ds*. If there are moving parts in the system and if the system geometry is complex, it may not be possible to do this. We start by using the fact that $(\nabla \cdot \rho v) = 0$ at steady state so that

$$E_c = -\int_V p(\nabla \cdot \mathbf{v})\, dV = +\int_V \frac{p}{\rho}(\mathbf{v} \cdot \nabla\rho)\, dV \tag{7.8-9}$$

Then when use the assumption that the pressure and density are constant over the cross section to write approximately:

$$E_c \approx \int_1^2 \frac{p}{\rho}\left(v\frac{d\rho}{ds}\right)S(s)ds \tag{7.8-10}$$

Even though $\rho, v,$ and S are functions of the streamline coordinate s, their product, $w = \rho v S$, is a constant for steady-state operation and hence may be taken outside the integral. This gives

$$E_c \approx w\int_1^2 \frac{p}{\rho^2}\frac{d\rho}{ds}ds = -w\int_1^2 p\frac{d}{ds}\left(\frac{1}{\rho}\right)ds \tag{7.8-11}$$

Then integration by parts can be performed:

$$E_c \approx -w\left[\frac{p}{\rho}\bigg|_1^2 - \int_1^2 \frac{1}{\rho}\frac{dp}{ds}ds\right] = -w\Delta\left(\frac{p}{\rho}\right) + w\int_1^2 \frac{1}{\rho}dp \tag{7.8-12}$$