Group 5

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Prof. Ubaldo Córdova

Ivette Matos López

Carlos I. Medina García

Luis E. Medina Pérez

Myrsa M. Morales Lozada

Carlos R Morales Rodriguez

Isamar Moreno López

Hiram G. Menéndez Santiago

Audrey Michelli Rivera

Chapter 5: FLOW ANALYSIS USING CONTROL VOLUME (Munson)

5.1 Conservation of mass - The continuity equation

5.1.1 Derivation of the continuity equation

A system is defined as a collection of unchanging contents, so the conservation of mass principle for a system is simply stated as

time rate of change of the system mass = 0

or

$$\frac{DM_{sys}}{Dt} = 0 \qquad (5.1)$$

where the system mass, M_{sys}, is more generally expressed as

$$M_{sys} = \int_{sys} \rho \, d\forall \qquad (5.2)$$

and the integration is over the volume of the system. In words, Eq. states that the system mass is equal to the sum of all the density-volume element products for the contents of the system.

time rate of change of the mass of the coincident system	=	time rate of change of the mass of the contents of the coincident control volume	+	net rate of flow of mass through the control surface

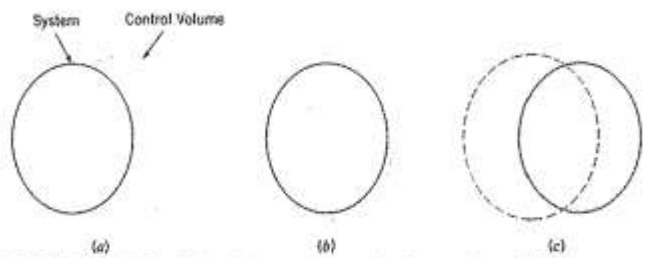


FIGURE 5.1 System and control volume at three different instances of time. (*a*) System and control volume at time $t - \delta t$. (*b*) System and control volume at time t, coincident condition. (*c*) System and control volume at time $t + \delta t$.

When a flow is steady, all field properties including density remain constant with time and the time rate of change of the mass of the contents of the control volume is zero.

$$\frac{\partial}{\partial t}\int_{\text{cv}} \rho \, d\mathcal{V} = 0$$

The integrand, $\mathbf{V} \cdot \hat{\mathbf{n}} \, dA$, in the mass flow rate integral represents the product of the component of velocity, V, perpendicular to the small portion of control surface and the differential area, dA. Thus, $\mathbf{V} \cdot \hat{\mathbf{n}} \, dA$, is the volume flow rate through dA and $\rho \mathbf{V} \cdot \hat{\mathbf{n}} \, dA$ is the mass flow rate through dA. Furthermore, the sign of the dot product $\mathbf{V} \cdot \hat{\mathbf{n}}$ is "+" for flow out of the control volume and "-"for flow into the control volume. When all of the differential quantities $\rho \mathbf{V} \cdot \hat{\mathbf{n}} \, dA$, are summed over the entire control surface the result is the net mass flow rate through the control surface

$$\int_{\text{cs}} \rho \mathbf{V} \cdot \hat{\mathbf{n}} \, dA = \sum \dot{m}_{\text{out}} - \sum \dot{m}_{\text{in}} \tag{5.4}$$

Where $\dot{m}$ is the mass flow rate.

The control volume expression for conservation of mass, continuity equation, for a fixed, nondeforming control volume is obtained by combining Eqs. 5.1, 5.2, and 5.3 to obtain

$$\frac{\partial}{\partial t}\int_{cv} \rho \, d\forall + \int_{cs} \rho \mathbf{V} \cdot \hat{\mathbf{n}} \, dA = 0 \tag{5.5}$$

An often used expression for mass flow rate through a section of control surface having area A is

$$\dot{m} = \rho Q = \rho A V \tag{5.6}$$

For incompressible flows, density is uniformly distributed over area A. For compressible flows, we will normally consider a uniformly distributed fluid density at each section of flow and allow density changes to occur only from section to section.

If the velocity is considered uniformly distributed over the section area, A, then

$$\overline{V} = \frac{\int_A \rho \mathbf{V} \cdot \hat{\mathbf{n}} \, dA}{\rho A} = V \tag{5.8}$$

5.1.2 Fixed, Nondeforming Control Volume

When the flow is steady, the time rate of change of the mass of the contents of the control volume

$$\frac{\partial}{\partial t}\int_{cv} \rho \, d\forall$$

is zero and the net amount of mass flow rate through the control surface is therefore also zero.

$$\sum \dot{m}_{out} - \sum \dot{m}_{in} = 0 \tag{5.9}$$

If the steady flow is also incompressible,

$$\sum Q_{out} - \sum Q_{in} = 0 \qquad \textbf{(5.10)}$$

When the flow is uniformly distributed over the opening in the control surface,

$$\dot{m} = \rho A V$$

For steady flow involving only one stream of a specific fluid flowing through the control volume,

$$\dot{m} = \rho_1 A_1 \overline{V}_1 = \rho_2 A_2 \overline{V}_2 \qquad \textbf{(5.12)}$$

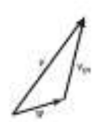
For steady flow involving more than one stream of a specific fluid or more than one specific fluid flowing through the control volume,

$$\sum \dot{m}_{in} = \sum \dot{m}_{out}$$

5.1.3 Moving, Nondeforming Control Volume

It is sometimes necessary to use a nondeforming control volume attached to a moving reference frame. The relative velocity, W, is the fluid velocity seen by an observer moving with the control volume. The control volume velocity, V_{CV}, is the velocity of the control volume as seen from a fixed coordinate system. The absolute velocity, V, is the fluid velocity seen by a stationary observer in a fixed coordinate system.

$$\mathbf{V = W + V_{CV}} \qquad \textbf{(5.14)}$$



For a system and a moving, nondeforming control volume that is coincident at an instant of time, the Reynolds transport theorem leads to

$$\frac{DM_{sys}}{Dt} = \frac{\partial}{\partial t}\int_{cv} \rho \, d\forall + \int_{cs} \rho \mathbf{W} \cdot \hat{\mathbf{n}} \, dA \qquad (5.15)$$

From Eqs. 5.1 and 5.15, we can get the control volume expression for conservation of mass (the continuity equation)

$$\boxed{\frac{\partial}{\partial t}\int_{cv} \rho \, d\forall + \int_{cs} \rho \mathbf{W} \cdot \hat{\mathbf{n}} \, dA = 0} \qquad (5.16)$$

5.1.4 Deforming Control Volume

A deforming control volume involves changing volume size and control surface movement. Thus, the Reynolds transport theorem for a moving control volume can be used for this case, and Eqs. 4.23 and 5.1 lead to

$$\boxed{\frac{DM_{sys}}{Dt} = \frac{\partial}{\partial t}\int_{cv} \rho \, d\forall + \int_{cs} \rho \mathbf{W} \cdot \hat{\mathbf{n}} \, dA = 0} \qquad (5.17)$$

The time rate of change term in Eq. 5.17,

$$\frac{\partial}{\partial t}\int_{cv} \rho \, d\forall$$

is usually nonzero and must be carefully evaluated because the extent of the control volume varies with time. The mass flow rate term in Eq. 5.17,

$$\int_{cs} \rho \mathbf{W} \cdot \hat{\mathbf{n}} \, dA$$

must be determine with W-relative velocity. Since the control volume is deforming, the control surface velocity is not necessarily uniform and identical to the V_{CS}. For the deforming control volume,

$$\mathbf{V} = \mathbf{W} + \mathbf{V}_{CS}$$

where V_{CS} is the velocity of the control surface as seen by a fixed observer.

5.2 Newton's Second Law – The Linear Momentum and Moment-of-Momentum Equations

5.2.1 Derivation of the Linear Momentum Equation

Newton's second law of motion deals with system momentum and forces.

time rate of change of the linear momentum of the system	=	sum of external forces acting of the system

Momentum is the product of mass times velocity, and for a small particle of mass Newton's law becomes

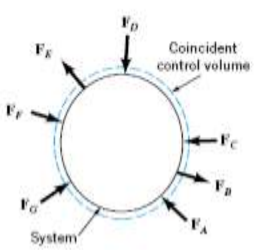
$$\frac{D}{Dt}\int_{sys} \mathbf{V}\rho \, d\forall = \sum \mathbf{F}_{sys}$$

Any system that follows the previous statement is called *inertial*. In case that a control volume coincides with a system at an instant of time, the forces acting on the system equal the forces acting on the contents of the coincident control volume (see Fig. 5.2),

$$\sum \mathbf{F}_{sys} = \sum \mathbf{F}_{\text{contents of the coincident control volume}}$$

For a system and the contents of a coincident control volume that is fixed and nondeforming, the Reynolds transport theorem allows us to conclude that

$$\frac{D}{Dt}\int_{sys} \mathbf{V}\rho \, d\forall = \frac{\partial}{\partial t}\int_{cv} \mathbf{V}\rho \, d\forall + \int_{cs} \mathbf{V}\rho\mathbf{V} \cdot \hat{\mathbf{n}} \, dA$$



■ FIGURE 5.2 External forces acting on system and coincident control volume.

An appropriate mathematical statement of Newton's second law of motion for a control volume that is fixed and nondeforming is the **linear momentum equation.**

$$\frac{\partial}{\partial t}\int_{\text{cv}} \mathbf{V}\rho \, d\forall + \int_{\text{cs}} \mathbf{V}\rho\mathbf{V}\cdot\hat{\mathbf{n}}\, dA = \sum \mathbf{F}_{\substack{\text{contents of the}\\ \text{control volume}}}$$

The forces involved in the previous equation are body and surface forces that act on what is contained in the control volume.

5.2.2 Application of the Linear Momentum Equation

- The linear momentum equation for an inertial control volume is a vector equation:

$$\frac{\partial}{\partial t}\int_{\text{cv}} \mathbf{V}\rho \, d\forall + \int_{\text{cs}} \mathbf{V}\rho\mathbf{V}\cdot\hat{\mathbf{n}}\, dA = \sum \mathbf{F}_{\substack{\text{contents of the}\\ \text{control volume}}}$$

- The linear momentum problems that involved stationary and non deforming control volumes which are thus inertial because there is no acceleration. A non deforming control volume translating in a straight line at constant speed is also inertial because there is no acceleration.
- For a system and an inertial, moving, non deforming control volume that are both coincident at an instant of time, the Reynolds transport theorem leads to:

$$\frac{D}{Dt}\int_{\text{sys}} \mathbf{V}\rho \, d\forall = \frac{\partial}{\partial t}\int_{\text{cv}} \mathbf{V}\rho \, d\forall + \int_{\text{cs}} \mathbf{V}\rho\mathbf{W}\cdot\hat{\mathbf{n}}\, dA$$

- When we combine :

$$\frac{D}{Dt}\int_{\text{sys}} \mathbf{V}\rho \, d\forall = \frac{\partial}{\partial t}\int_{\text{cv}} \mathbf{V}\rho \, d\forall + \int_{\text{cs}} \mathbf{V}\rho\mathbf{W}\cdot\hat{\mathbf{n}}\, dA$$

With equations:

$$\frac{D}{Dt}\int_{\text{sys}} \mathbf{V}\rho \, d\forall = \sum \mathbf{F}_{\text{sys}}$$

&

$$\sum \mathbf{F}_{\text{sys}} = \sum \mathbf{F}_{\substack{\text{contents of the}\\ \text{coincident control volume}}}$$

We get:

$$\frac{\partial}{\partial t}\int_{cv}\mathbf{V}\rho\, d\Psi + \int_{cs}\mathbf{V}\rho\mathbf{W}\cdot\hat{\mathbf{n}}\, dA = \sum \mathbf{F}_{\substack{\text{contents of the}\\ \text{control volume}}}$$

- When the equation relating absolute, relative, and control volume velocities is used with the last equation, the result is:

$$\frac{\partial}{\partial t}\int_{cv}(\mathbf{W}+\mathbf{V}_{cv})\rho\, d\Psi + \int_{cs}(\mathbf{W}+\mathbf{V}_{cv})\rho\mathbf{W}\cdot\hat{\mathbf{n}}\, dA = \sum \mathbf{F}_{\substack{\text{contents of the}\\ \text{control volume}}}$$

- For a constant control volume velocity. $\mathbf{V}_{\mathbf{cv}}$, and steady flow in the control volume reference frame,

$$\frac{\partial}{\partial t}\int_{cv}(\mathbf{W}+\mathbf{V}_{cv})\rho\, d\Psi = 0$$

- Also, for this inertial, non deforming control volume:

$$\int_{cs}(\mathbf{W}+\mathbf{V}_{cv})\rho\mathbf{W}\cdot\hat{\mathbf{n}}\, dA = \int_{cs}\mathbf{W}\rho\mathbf{W}\cdot\hat{\mathbf{n}}\, dA + \mathbf{V}_{cv}\int_{cs}\rho\mathbf{W}\cdot\hat{\mathbf{n}}\, dA$$

- Finally we conclude that the linear momentum equation for an inertial, moving, non deforming control volume that involves steady flow is:

$$\int_{cs}\mathbf{W}\rho\mathbf{W}\cdot\hat{\mathbf{n}}\, dA = \sum \mathbf{F}_{\substack{\text{contents of the}\\ \text{control volume}}}$$

5.2.3 Derivation of the Moment-of-Momentum Equation[2]

When torques are important, the moment of momentum equation is often more convenient to use than the linear momentum equation. By forming the moment of the linear momentum and the resultant force of each particle in the fluid with respect to a point in an inertial coordinate system, a **moment-of-momentum equation** is developed.

$$\frac{D}{Dt}(\mathbf{V}\rho\,\delta\Psi) = \delta\mathbf{F}_{\text{particle}}$$

5.3 First Law of Thermodynamics – The Energy Equation

5.3.1 Derivation of the Energy Equation

The **first law of thermodynamics** for a system is, in words

time rate of increase of the total stored energy of the system	=	net time rate of energy addition bye heat transfer into the system	+	net time of energy addition by work transfer into the system

In symbolic form, this statement is

$$\frac{D}{Dt}\int_{\text{sys}} e\rho \, d\forall = (\dot{Q}_{\substack{\text{net}\\\text{in}}} + \dot{W}_{\substack{\text{net}\\\text{in}}})_{\text{sys}}$$

The total energy per unit mass for each particle in the system, e, is related to the internal energy per unit mass, kinetic energy per unit mass, and the potential energy per unit mass, by the equation

$$e = \check{u} + \frac{V^2}{2} + gz$$

The net **rate of heat transfer** into the system is denoted with $\dot{Q}_{\text{net in}}$, and the net rate of work transfer into the system is labeled $\dot{W}_{\text{net in}}$. Heat transfer and work transfer are (+) going into the system and (-) coming out. For the control volume that is coincident with the system at an instant of time

$$(\dot{Q}_{\substack{\text{net}\\\text{in}}} + \dot{W}_{\substack{\text{net}\\\text{in}}})_{\text{sys}} = (\dot{Q}_{\substack{\text{net}\\\text{in}}} + \dot{W}_{\substack{\text{net}\\\text{in}}})_{\substack{\text{coincident}\\\text{control volume}}}$$

Furthermore, for the system and the contents of the coincident control volume that is fixed and nondeforming, the Reynolds transport theorem allows us to conclude that

$$\frac{D}{Dt}\int_{\text{sys}} e\rho \, d\forall = \frac{\partial}{\partial t}\int_{\text{cv}} e\rho \, d\forall + \int_{\text{cs}} e\rho \mathbf{V}\cdot\hat{\mathbf{n}} \, dA$$

Combining the previous equations we get the control volume formula for the first law of thermodynamics:

$$\frac{\partial}{\partial t}\int_{cv} e\rho \, d\forall + \int_{cs} e\rho \mathbf{V} \cdot \hat{\mathbf{n}} \, dA = (\dot{Q}_{\underset{in}{net}} + \dot{W}_{\underset{in}{net}})_{cv}$$

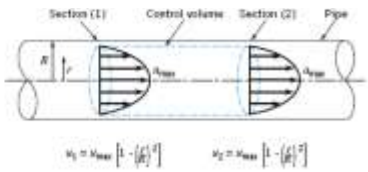


FIGURE 5.6 Simple, fully developed pipe flow.

For the control volume of Fig. 5.6, the fluid particle velocity is zero everywhere on the wetted inside surface of the pipe. Thus, no tangential stress work is transferred across that portion of the control surface. Furthermore, where fluid crosses the control surface, the tangential stress force is perpendicular to the velocity and therefore tangential stress work transfer is also zero there.

Taking into account the equation for total stored energy we can obtain a more general expression for the ***energy equation***:

$$\frac{\partial}{\partial t}\int_{cv} e\rho \, d\forall + \int_{cs}\left(\check{u} + \frac{p}{\rho} + \frac{V^2}{2} + gz\right)\rho \mathbf{V} \cdot \hat{\mathbf{n}} \, dA = \dot{Q}_{\underset{in}{net}} + \dot{W}_{\underset{net\ in}{shaft}}$$

5.3.2 Application of the Energy Equation

Energy equation:

$$\frac{\partial}{\partial t}\int_{cv} e\rho \, d\forall + \int_{cs} (u + \frac{p}{\rho} + \frac{V^2}{2} + gz)\rho \boldsymbol{V} \cdot \boldsymbol{n} \, dA$$

The term $\frac{\partial}{\partial t}\int_{cv} e\rho \, d\forall$ in the energy equation represent the time rate of change of the total stored energy, e, of the control volume. When the flow is steady and in the mean when the flow is steady in the mean (cyclical) this term is zero. The equation energy equation is reduced to:

$$\int_{cs} (u + \frac{p}{\rho} + \frac{V^2}{2} + gz) \, \rho \boldsymbol{V} \cdot \boldsymbol{n} \, dA$$

In the reduced energy equation the integrand can be nonzero only where fluid crosses the control surface ($\boldsymbol{V} \cdot \boldsymbol{n} \neq 0$). Otherwise, $\boldsymbol{V} \cdot \boldsymbol{n}$ is zero and the integrand is zero for that portion of the control surface.

The flow in any fluid machine that involves shaft work is unsteady within that machine. Most often, shaft work is associated with flow that is unsteady in a recurring or cyclical way. For flow that is one-dimensional, cyclical and involves only one stream of fluid entering and leaving the control volume, the *energy equation* can be simplified using the following equations:

- $\sum m_{out}$ - $\sum m_{in}$ = 0
- $\int_{cs} (u + \frac{p}{\rho} + \frac{V^2}{2} + gz)\, \rho \boldsymbol{V} \cdot \boldsymbol{n}\, dA =$

$$\left(u + \frac{p}{\rho} + \frac{V^2}{2} + gz\right)_{out} m_{out} - \left(u + \frac{p}{\rho} + \frac{V^2}{2} + gz\right)_{in} m_{in}$$

And we obtain the *one-dimensional energy equation for steady in the mean flow*:

$$m\left[u_{out} - u_{in} + \left(\frac{p}{\rho}\right)_{out} - \left(\frac{p}{\rho}\right)_{in} + \frac{V_{out}^2 - V_{in}^2}{2} + g(z_{out} - z_{in})\right] = Q_{\substack{net \\ in}} - W_{\substack{shaft \\ net\ in}}$$

- This equation is valid for incompressible and compressible flows.
- Enthalpy, h , is a fluid property is defined as: $h = u + \frac{p}{\rho}$

We can use de enthalpy definition to rewrite the *one-dimensional energy equation for steady in the mean flow as:*

$$m\left[h_{out} - h_{in} + \frac{V_{out}^2 - V_{in}^2}{2} + g(z_{out} - z_{in})\right] = Q_{\substack{net \\ in}} - W_{\substack{shaft \\ net\ in}}$$

This equation is often used for solving compressible flow problems.

5.3.3 Comparison of the Energy Equation with the Bernoulli Equation

The shaft power,W_{shaft}, is zero if the flow is steady throughout the control volume. If in addition to being steady, the flow in incompressible, we obtain:

$$m\left[u_{out} - u_{in} + \left(\frac{p}{\rho}\right)_{out} - \left(\frac{p}{\rho}\right)_{in} + \frac{V_{out}^2 - V_{in}^2}{2} + g(z_{out} - z_{in})\right] = Q_{\substack{net \\ in}}$$

Dividing the above equation by the mass flow rate;m, and rearranging terms we obtain:

$$\frac{p_{out}}{\rho} + \frac{V_{out}^2}{2} + gz_{out} = \frac{p_{in}}{\rho} + \frac{V_{in}^2}{2} + gz_{in} - (u_{out} - u_{in} - q_{net\ in})$$

Where

- $q_{net\ in} = \frac{Q_{net\ in}}{m}$ = the heat transfer rate per mass flowrate, or heat transfer per unit mass.

If the steady, incompressible flow we are considering also involves negligible viscous effects, then the Bernoulli equation;

$$p + \frac{1}{2}\rho V^2 + \gamma z = constant\ along\ streamline$$

describe what happens between two sections in the flow as:

$$p_{out} + \frac{\rho V_{out}^2}{2} + \gamma z_{out} = p_{in} + \frac{\rho V_{in}^2}{2} + \gamma z_{in}$$

Where $\gamma = \rho g$ is the specific gravity of the fluid. Now we want the above ecuation in terms of energy per unit mass:

$$\frac{p_{out}}{\rho} + \frac{V_{out}^2}{2} + \gamma z_{out} = \frac{p_{in}}{\rho} + \frac{V_{in}^2}{2} + \gamma z_{in}$$

In addition, we can conclude that $(u_{out} - u_{in} - q_{net\ in})$ is the loss of useful energy that occurs in an incompressible fluid because of friction.

$$\frac{p_{out}}{\rho} + \frac{V_{out}^2}{2} + \gamma z_{out} = \frac{p_{in}}{\rho} + \frac{V_{in}^2}{2} + \gamma z_{in} - loss$$

Important problems involves one-dimensional, incompressible, steady in the mean flow with friction and shaft work. For this kind of flow *the one-dimensional energy equation for steady in the mean* flow becomes:

$$m\left[u_{out} - u_{in} + \frac{p_{out}}{\rho} - \frac{p_{in}}{\rho} + \frac{V_{out}^2 - V_{in}^2}{2} + g(z_{out} - z_{in})\right] = Q_{\substack{net \\ in}} - W_{\substack{shaft \\ net\ in}}$$

Dividing the previous equation by mass flow rate and using the work per unit mass, $w_{\substack{shaft \\ net\ in}} = W_{\substack{shaft \\ net\ in}}/m$, we obtain:

$$\frac{p_{out}}{\rho} + \frac{V_{out}^2}{2} + gz_{out} = \frac{p_{in}}{\rho} + \frac{V_{in}^2}{2} + gz_{in} + w_{\substack{shaft \\ net\ in}} - (u_{out} - u_{in} - q_{net\ in})$$

If the flow is steady we can rewrite the above equation as:

$$\frac{p_{out}}{\rho} + \frac{V_{out}^2}{2} + gz_{out} = \frac{p_{in}}{\rho} + \frac{V_{in}^2}{2} + gz_{in} + w_{\substack{shaft \\ net\ in}} - loss$$

This is a form of energy equation for steady in the mean flow that is often used for incompressible problems.

When the shaft work is into the control volume a larger amount of loss will result in **more** shaft work being required for the same rise in available energy. If the shaft work is out of the control volume a larger loss will result in **less** shaft work out for the same drop in available energy.

Now the previous equation is multiplied by fluid density we obtain:

$$p_{out} + \frac{\rho V_{out}^2}{2} + \gamma z_{out} = p_{in} + \frac{\rho V_{in}^2}{2} + \gamma z_{in} + \rho w_{\substack{shaft \\ net\ in}} - \rho(loss)$$

If we divide by the acceleration of gravity instead of multiply by density we obtain:

$$\frac{p_{out}}{\gamma} + \frac{V_{out}^2}{2g} + z_{out} = \frac{p_{in}}{\gamma} + \frac{V_{in}^2}{2g} + z_{in} + h_s - h_L$$

Where $h_s = \frac{w_{shaft\ net\ in}}{g} = \frac{W_{shaft\ net\ in}}{mg} = \frac{W_{shaft\ net\ in}}{\gamma Q} = shaft\ head$

and $h_L = \frac{loss}{g} = head\ loss$

The turbine head is written as:

$$h_T = -(h_s + h_L)_T$$

When a pump is in the control volume:

$$h_P = (h_s - h_L)_P$$

5.4 Second Law of Thermodynamics- Irreversible Flow

The second law of thermodynamics affords us with a means to formalize the inequality

$$\check{u}_2 - \check{u}_1 - q_{\underset{\text{in}}{\text{net}}} \geq 0$$

for steady, incompressible, one-dimensional flow with friction.

5.4.1 Semi-infinitesimal Control Volume Statement of the Energy Equation

If we apply the one-dimensional, steady flow energy equation to the contents of a control volume that is infinitesimally thin as illustrated in Fig 5.8, the result is

$$\dot{m}\left[d\check{u} + d\left(\frac{p}{\rho}\right) + d\left(\frac{V^2}{2}\right) + g\,(dz)\right] = \delta\dot{Q}_{\underset{\text{in}}{\text{net}}}$$

For all pure substances including common engineering working fluids, such as air, water, oil, and gasoline, the following relationship is valid

$$T\,ds = d\check{u} + pd\left(\frac{1}{\rho}\right)$$

where T is the absolute temperature and s is the *entropy* per unit mass.

Combining

$$\dot{m}\left[T\,ds - pd\left(\frac{1}{\rho}\right) + d\left(\frac{p}{\rho}\right) + d\left(\frac{V^2}{2}\right) + g\,dz\right] = \delta\dot{Q}_{\underset{\text{in}}{\text{net}}}$$

or, dividing through by $\dot{m}$ and letting $\delta q_{\underset{\text{in}}{\text{net}}} = \delta\dot{Q}_{\underset{\text{in}}{\text{net}}}/\dot{m}$,

$$\frac{dp}{\rho} + d\left(\frac{V^2}{2}\right) + g\,dz = -(T\,ds - \delta q_{\underset{\text{in}}{\text{net}}})$$

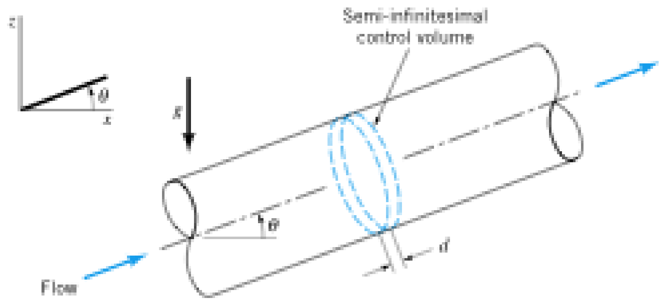


Figure 5.8

5.4.2 Semi-infinitesimal Control Volume Statement of the Second Law of Thermodynamics

A general statement of the second law of thermodynamics is

$$\frac{D}{Dt}\int_{\text{sys}} s\rho \, d\mathcal{V} \geq \sum\left(\frac{\delta \dot{Q}_{\substack{\text{net}\\ \text{in}}}}{T}\right)_{\text{sys}}$$

the time rate of increase of the entropy of a system ≥ sum of the ratio of net heat transfer rate into system to absolute temperature for each particle of mass in the system receiving heat from surroundings

If the right-hand side of the above statement is identical for the system and control volume at the instant when system and control volume are coincident;

$$\sum\left(\frac{\delta \dot{Q}_{\substack{\text{net}\\ \text{in}}}}{T}\right)_{\text{sys}} = \sum\left(\frac{\delta \dot{Q}_{\substack{\text{net}\\ \text{in}}}}{T}\right)_{\text{cv}}$$

Using the Reynolds transport theorem

$$\frac{D}{Dt}\int_{\text{sys}} s\rho \, d\mathcal{V} = \frac{\partial}{\partial t}\int_{\text{cv}} s\rho \, d\mathcal{V} + \int_{\text{cs}} s\rho \mathbf{V}\cdot\hat{\mathbf{n}} \, dA$$

For a fixed, nondeforming control volume, above three equations are combined to give

$$\frac{\partial}{\partial t}\int_{\text{cv}} s\rho \, d\mathcal{V} + \int_{\text{cs}} s\rho \mathbf{V}\cdot\hat{\mathbf{n}} \, dA \geq \sum\left(\frac{\delta \dot{Q}_{\substack{\text{net}\\ \text{in}}}}{T}\right)_{\text{cv}}$$

At any instant for steady flow

$$\frac{\partial}{\partial t}\int_{cv} s\rho \, d\forall = 0$$

If the flow consists of only one stream through the control volume and if the properties are uniformly distributed (one-dimensional flow),

$$\dot{m}(s_{out} - s_{in}) \geq \sum \frac{\delta \dot{Q}_{\substack{net\\in}}}{T}$$

For the infinitesimally thin control volume of Fig. 5.8,

$$\dot{m}\, ds \geq \sum \frac{\delta \dot{Q}_{\substack{net\\in}}}{T}$$

If all of the fluid in the infinitesimally thin control volume is considered as being at a uniform temperature, *T*, then

$$T\, ds \geq \delta q_{\substack{net\\in}}$$

5.4.3 Combination of the Equations of the First and Second Laws of Thermodynamics

This equation represents the extent of loss of useful or available energy which occurs because of irreversible flow phenomena including viscous effects**.**

$$-\left[\frac{dp}{\rho} + d\left(\frac{V^2}{2}\right) + g\, dz\right] \geq 0$$

The equality is for any steady, reversible (frictionless) flow and the inequality is for all steady, irreversible (friction) flows. Thus the equation can be expressed as,

$$-\left[\frac{dp}{\rho} + d\left(\frac{V^2}{2}\right) + g\, dz\right] = \delta(\text{loss}) = (T\, ds - \delta q_{\substack{net\\in}})$$

The irreversible flow loss is zero for a frictionless flow and greater than zero for a flow with frictional effects. For steady frictionless flow, Newton's second law of motion and the first and second laws of thermodynamics lead to the same differential equation,

$$\frac{dp}{\rho} + d\left(\frac{V^2}{2}\right) + g\,dz = 0$$

If some shaft work is involved,

$$-\left[\frac{dp}{\rho} + d\left(\frac{V^2}{2}\right) + g\,dz\right] = \delta(\text{loss}) - \delta w_{\substack{\text{shaft}\\\text{net in}}}$$

5.4.4 Application of the Loss Form of the Energy Equation

Steady flow along a path line in an incompressible and frictionless flow field provides a simple application of the loss form of the energy equation;

$$\frac{p_2}{\rho} + \frac{V_2^2}{2} + gz_2 = \frac{p_1}{\rho} + \frac{V_1^2}{2} + gz_1$$

which is identical to the Bernoulli equation. If the frictionless and steady path line flow of the fluid particle considered above was compressible,

$$\int_1^2 \frac{dp}{\rho} + \frac{V_2^2}{2} + gz_2 = \frac{V_1^2}{2} + gz_1$$

Relating fluid density, ρ, and pressure p,

$$\frac{p}{\rho^k} = \text{constant}$$

here $k = c_p/c_v$ is the ratio of gas specific heats, c_p and c_v, which are properties of the fluid. Substituting we get,

$$\int_1^2 \frac{dp}{\rho} = \frac{k}{k-1}\left(\frac{p_2}{\rho_2} - \frac{p_1}{\rho_1}\right)$$

Using both above integrates we get,

$$\frac{k}{k-1}\frac{p_2}{\rho_2} + \frac{V_2^2}{2} + gz_2 = \frac{k}{k-1}\frac{p_1}{\rho_1} + \frac{V_1^2}{2} + gz_1$$