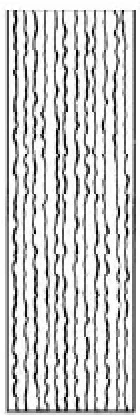
Topic 8: Flow in Packed Beds

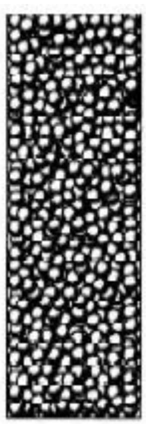
Class Notes - Chapter 6.4 (Transport Phenomena)

Friction factor correlations are available for a variety of systems. One complex system of considerable interest in chemical engineering is the packed column. There are two approaches for developing friction factor expressions for packed columns:

- Packed column visualized as a bundle of tangled tubes of variable cross section (more successful theory).



- Packed column regarded as a collection of submerged objects.



The pressure drop is obtained by adding up the resistances of the submerged objects (above figure).

Assumptions

- Packing is statistically uniform, so that there is no "channeling".
- Diameter of the packing particles is small in comparison to the diameter of the column in which the packing is contained.
- Column diameter uniform.

Definition of friction factor for the packed column

$$f = \frac{1}{4}\left(\frac{D_p}{L}\right)\left(\frac{\mathcal{P}_0 - \mathcal{P}_L}{\frac{1}{2}\rho v_0^2}\right)$$

Where;
L=length of the packed column
D_p= effective particle diameter
v_0 =superficial velocity

$$v_0 = w/\rho S$$

w= volume flow rate
ρS= empty column cross section

Correlations

Pressure drop ***through a representative tube*** in the tube bundle model:

$$\mathcal{P}_0 - \mathcal{P}_L = \frac{1}{2}\rho\langle v\rangle^2\left(\frac{L}{R_h}\right)f_{\text{tube}}$$

In which ***the friction factor for a single tube is a function of the Reynolds number***:

$$\text{Re}_h = 4R_h\langle v\rangle\rho/\mu$$

By substituting the pressure drop into the friction factor correlation we get:

$$f = \frac{1}{4}\frac{D_p}{R_h}\frac{\langle v\rangle^2}{v_0^2}f_{\text{tube}} = \frac{1}{4\epsilon^2}\frac{D_p}{R_h}f_{\text{tube}}$$

Where;
$\epsilon^2 = \left(\frac{v_0}{<v>}\right)^2$ *= A/S*
ϵ = void fraction = fraction of space in the column not occupied by the packing
A= available flow area
S= empty column area

The hydraulic radius can be expressed in terms of the void fraction and the wetted surface area per unit volume of bed as follows:

$$R_h = \left(\frac{\text{cross section available for flow}}{\text{wetted perimeter}}\right)$$
$$= \left(\frac{\text{volume available for flow}}{\text{total wetted surface}}\right)$$
$$= \frac{\left(\frac{\text{volume of voids}}{\text{volume of bed}}\right)}{\left(\frac{\text{wetted surface}}{\text{volume of bed}}\right)} = \frac{\varepsilon}{a}$$

Where;
a= wetted surface area

and is related to the “specific surface” by:

$$a_v = \frac{a}{1-\varepsilon}$$

a_v is the “specific surface” (total particles surface per volume of particles) and is used to define the mean particle diameter by:

$$D_p = \frac{6}{a_v}$$

and

$$R_h = D_p\varepsilon/6(1-\varepsilon).$$

By substituting this equation (hydraulic radius) into the ***friction factor correlation*** we get;

$$f = \frac{3}{2}\left(\frac{1-\varepsilon}{\varepsilon^3}\right)f_{\text{tube}}$$

We now adapt this result to laminar and turbulent flows by inserting appropriate expressions f_{tube}.

A. Laminar flow

f_{tube}= 16/Re_h

Experimentally f_{tube}=100/(3Re_h)

By substituting this equation (f_{tube}) into the friction factor correlation we get;

$$f = \frac{(1-\varepsilon)^2}{\varepsilon^3}\frac{75}{(D_pG_0/\mu)}$$

Where;
G_0=ρv_0= mass flux to the system and
1-ε= (2/3) D_pG_0/μ

When this expression for *f* is substituted into its initial expression we get

$$\frac{\mathscr{P}_0 - \mathscr{P}_L}{L} = 150\left(\frac{\mu v_0}{D_p^2}\right)\frac{(1-\varepsilon)^2}{\varepsilon^3}$$

This is known as the ***Blake-Kozeny equation.***

The last two equations do not apply to any system. They are physically limited for:

- $\varepsilon<0.5$
- $D_pG_0/[\mu(1-\varepsilon)] <10$

B. Highly Turbulent Flow

For highly turbulent flow in tubes with any appreciable roughness, ***the friction factor is a function of the roughness only, and is independent of the Reynolds number.***

F_{tube}= 7/12

By substituting this equation into the friction factor correlation we get;

$$f = \frac{7}{8}\left(\frac{1-\varepsilon}{\varepsilon^3}\right)$$

When this expression for *f* is substituted into its initial expression we get

$$\frac{\mathscr{P}_0 - \mathscr{P}_L}{L} = \frac{7}{4}\left(\frac{\rho v_0^2}{D_p}\right)\frac{1-\varepsilon}{\varepsilon^3}$$

This is the ***Burke-Plummer equation*** and is valid for:

$$(D_pG_0/\mu(1-\varepsilon)) > 1000.$$

Note that the dependence on the void fraction is different from that for laminar flow.

C. Transition Region

By superposing the two expressions for pressure drop for the case A and B we get

$$\frac{\mathscr{P}_0 - \mathscr{P}_L}{L} = 150\left(\frac{\mu v_0}{D_p^2}\right)\frac{(1-\varepsilon)^2}{\varepsilon^3} + \frac{7}{4}\left(\frac{\rho v_0^2}{D_p}\right)\frac{1-\varepsilon}{\varepsilon^3}$$

For very small v_0 this simplifies to the Blake-Kozeny equation and for very large v_0 to the Burke-Plummer equation.

The above equation may be rearranged to form dimensionless groups:

$$\left(\frac{(\mathscr{P}_0 - \mathscr{P}_L)\rho}{G_0^2}\right)\left(\frac{D_p}{L}\right)\left(\frac{\varepsilon^3}{1-\varepsilon}\right) = 150\left(\frac{1-\varepsilon}{D_p G_0/\mu}\right) + \frac{7}{4}$$

This is the ***Ergun equation.*** The Ergun equation is but one of many that have been proposed for describing packed columns.

The Tallmadge equation is reported to give good agreement with experimental data over the range

$$0.1 < (D_p G_0/\mu(1-\varepsilon)) < 10^5$$

Tallmadge equation:

$$\left(\frac{(\mathscr{P}_0 - \mathscr{P}_L)\rho}{G_0^2}\right)\left(\frac{D_p}{L}\right)\left(\frac{\varepsilon^3}{1-\varepsilon}\right) = 150\left(\frac{1-\varepsilon}{D_p G_0/\mu}\right) + 4.2\left(\frac{1-\varepsilon}{D_p G_0/\mu}\right)^{1/6}$$

Problems

6A-6) **Estimation of void fraction of a packed column.** A tube of 146 sq in cross section and 73 in height is packed with spherical particles of diameter 2mm. When a pressure difference of 158 psi is maintained across the column, a 60% aqueous sucrose solution at 20 C flows through the bed at a rate of 244 lb/min. At this temperature, the viscosity of the solution is 56.5 cp and its density is 1.2865g/cm^3. What is the void fraction of the bed? Discuss the usefulness of this method of obtaining the void fraction.

Data:

D = 146 in^2 $(\frac{2.54cm}{1\ in})$^2 = 941.93 cm^2

L= 73 in $(\frac{2.54\ cm}{1\ in})$ = 185.42 cm

D_p = 2 x 10^3 m $(\frac{100cm}{1\ m})$ = 0.2 cm

ΔP = 158 psi (6.8947 x 10^4) = 1.09 x 10^{7}g/ cm.s^2

w = 244 lb/min $\left(\frac{453.59\ g}{1\ lb}\right)\left(\frac{1\ min}{60\ s}\right)$ = 1844.60 g/s μ= 56.5 cp

ρ= 1.2865 g/ cm^3

First we obtain the v_0

$$v_0 = w/\rho S$$

$$v_0 = \frac{1844.60\frac{g}{s}}{\left(1.2865\frac{g}{cm^3}\right)(941.93cm^2)} = \mathbf{1.522cm/s}$$

Then we obtain void fraction

$$\frac{\varepsilon^3}{(1-\varepsilon)^2} = \frac{150\mu L v0}{Dp^2 \Delta P}$$

$$\frac{\varepsilon^3}{(1-\varepsilon)^2} = \frac{150(0.565)(185.42)(1.522)}{(0.2)^2(1.09\ x\ 10^7)}$$

$$\frac{\varepsilon^3}{(1-\varepsilon)^2} = .05486$$

ε = **0.2997 =0.30**

6A.9) Flow of gas through a packed column. A horizontal tube with a diameter 4 in. and length 5.5 ft is packed with glass spheres of diameter 1/16 in., and the void fraction is 0.41. Carbon dioxide is to be pumped through the tube at 300K, at which temperature its viscosity is known to be 1.495 x 10^{-4} g/cm s. What will be the mass flow rate through the column when the inlet and outlet pressures are25 atm and 3 atm, respectively?

Data:

D= 4.0 in$(\frac{2.54 cm}{1\ in})$ = 10.16 cm p_0 = 25atm$\left(\frac{1.0133\ x\ 10^5}{1\ atm}\right)\left(\frac{1000\,g}{1\ kg}\right)\left(\frac{1\ m}{100 cm}\right)$= 2.533 x 10^7 g/cm.s^2

L= 5.5 ft $(\frac{12\ in}{1\ ft})(\frac{2.54\ cm}{1\ in})$ = 167.64 cm
μ = 1.495 x 10^{-4} g/cm.s

p_L = 3atm$\left(\frac{1.0133\ x\ 10^5}{1\ atm}\right)\left(\frac{1000\,g}{1\ kg}\right)\left(\frac{1\ m}{100 cm}\right)$= 3.0400 x 10^6 g/cm.s^2 ε = 0.41

D_p = 1/16 in$(\frac{2.54 cm}{1\ in})$= 0.15875cm

Equation:

$$\frac{\text{Po}-\text{PL}}{\text{L}} = 150\left(\frac{\mu vo}{Dp^2}\right)\left(\frac{(1-\varepsilon)^2}{(\varepsilon^3)}\right) + \frac{7}{4}\left(\frac{\rho vo^2}{Dp}\right)\left(\frac{1-\varepsilon}{\varepsilon^3}\right)$$

Applying differentiation:

$$-\frac{\text{dp}}{\text{dz}} = 150\left(\frac{\mu vo}{Dp^2}\right)\left(\frac{(1-\varepsilon)^2}{(\varepsilon^3)}\right) + \frac{7}{4}\left(\frac{\rho vo^2}{Dp}\right)\left(\frac{1-\varepsilon}{\varepsilon^3}\right)$$

And: $v_0 = G_0/\rho$

$$-\frac{\text{dp}}{\text{dz}} = 150\left(\frac{\mu Go}{\rho Dp^2}\right)\left(\frac{(1-\varepsilon)^2}{(\varepsilon^3)}\right) + \frac{7}{4}\left(\frac{Go^2}{\rho Dp}\right)\left(\frac{1-\varepsilon}{\varepsilon^3}\right)$$

Then we assume CO_2-is an ideal gas

$$\rho = \frac{PF}{RT}$$

Integrating

$$\int_{\text{po}}^{\text{pL}} \text{dp} = \int_0^L \left(150\left(\frac{\mu Go}{\rho Dp^2}\right)\left(\frac{(1-\varepsilon)^2}{(\varepsilon^3)}\right) + \frac{7}{4}\left(\frac{Go^2}{\rho Dp}\right)\left(\frac{1-\varepsilon}{\varepsilon^3}\right)\right) dz$$

$$-(\text{po}^2-\text{pL}^2) = 150\left(\left(\frac{\mu Go}{\rho Dp^2}\right)\left(\frac{(1-\varepsilon)^2}{(\varepsilon^3)}\right) + \frac{7}{4}\left(\frac{Go^2}{\rho Dp}\right)\left(\frac{1-\varepsilon}{\varepsilon^3}\right)\right) L$$

Multiplying by ρ we obtain:

$$-\rho(po^2 - pL^2) = 150\left(\left(\frac{\mu Go}{Dp^2}\right)\left(\frac{(1-\varepsilon)^2}{(\varepsilon^3)}\right) + \frac{7}{4}\left(\frac{Go^2}{Dp}\right)\left(\frac{1-\varepsilon}{\varepsilon^3}\right)\right)L$$

To do it simple, we do the calculation by terms

- First Term

$$150\left(\frac{\mu Go}{\rho Dp^2}\right)\left(\frac{(1-\varepsilon)^2}{(\varepsilon^3)}\right) L = 150\left(\frac{1.495\ x\ 10^{-4} Go}{0.15875^2}\right)\left(\frac{(1-0.41)^2}{(0.41^3)}\right)(167.64)$$

$$150\left(\frac{\mu Go}{\rho Dp^2}\right)\left(\frac{(1-\varepsilon)^2}{(\varepsilon^3)}\right) L = \mathbf{753.42\ Go}\left[\frac{\mathbf{g^2}}{\mathbf{s\ cm^4}}\right]$$

- Second Term

$$\frac{7}{4}\left(\frac{Go^2}{\rho Dp}\right)\left(\frac{1-\varepsilon}{\varepsilon^3}\right)L = \frac{7}{4}\left(\frac{Go^2}{0.15875}\right)\left(\frac{1-0.41}{0.41^3}\right)(167.64)$$

$$\frac{7}{4}\left(\frac{Go^2}{\rho Dp}\right)\left(\frac{1-\varepsilon}{\varepsilon^3}\right)L = \mathbf{15819.85\ Go^2}\left[\frac{\mathbf{g^2}}{\mathbf{s\ cm^4}}\right]$$

- Third Term

$$\rho(po - pL) = \frac{PF}{RT}(po^2 - pL^2)$$

$$\frac{PF}{RT}(po^2 - pL^2) = \frac{44.01}{(8.3145\ x10^7)300}\left((2.533x10^7)^2 - (3.0400x10^6)^2\right)$$

$$\frac{PF}{RT}(po^2 - pL^2) = \mathbf{1.116x10^6}\left[\frac{\mathbf{g^2}}{\mathbf{s\ cm^4}}\right]$$

Now we obtain:

$$\mathbf{-1.116x10^6 = 753.42\ Go + 15819.85\ Go^2}$$

Then we use the Quadratic Formula

$$Go = \frac{-753.42 \pm \sqrt{753.42^2 + 4(15819.85)(1.116x10^6)}}{2(15819.85)}$$

$$Go = \frac{-753.42 \pm 265744.95}{31639.7}$$

The positive root is:

$$Go = \mathbf{8.375}\left[\frac{\mathbf{g^2}}{\mathbf{s\ cm^4}}\right]$$

Then we obtain the mass flow rate w

$$w = \frac{\pi}{4}D^2\mathrm{Go}$$

$$w = \frac{\pi}{4}(10.16)^2(8.375)$$

$$w = \mathbf{679.0\frac{g}{s}}$$

References/Aditional Information

- BSL, Chapter # 6, Section # 4
- http://web2.clarkson.edu/projects/subramanian/ch301/notes/packfluidbed.pdf
- http://rothfus.cheme.cmu.edu/tlab/pbeds/projects/t4_s04/t4_s04.pdf

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